Radicals and Radical Functions - ANSWERS

RD1 Exercises

1. 7 **3.** not a real number **5.** 0.04 **7.** 4

9. 0.2 **11.** $\frac{1}{0.03}$ **13.** 0.2 **15.** not a real number

17. **a.** negative **b.** not a real number **c.** 0 **19.** 15 **21.** $|x|$

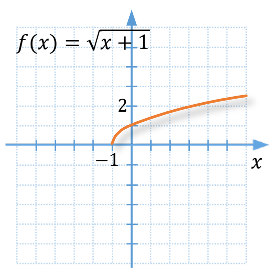
23. $9|x|$ **25.** $|x+3|$ **27.** $|x-2|$ **29.** -5

31. $-5a$ **33.** $5|x|$ **35.** $y-3$ **37.** $|2a-b|$

39. $|a+1|^3$ **41.** $-k^5$ **43.** 18.708 **45.** 1.710

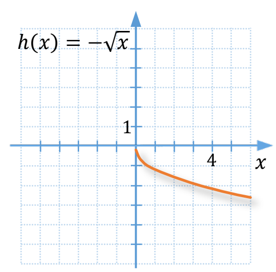
47. 8 **49.** 11 **51.** 50 **53.** 14 m by 7 m; 42 m

55. $D=[-1,\infty)$
range $=[0,\infty)$



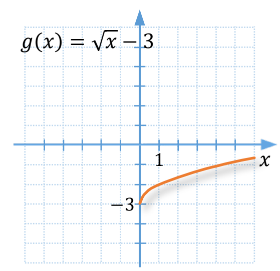
Translation: 1 step to the left

57. $D=[0,\infty)$
range $=(-\infty,0]$



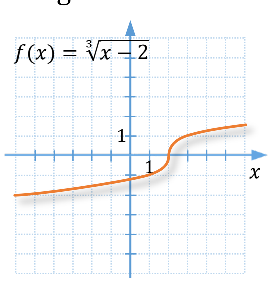
Reflection in x-axis

59. $D=[0,\infty)$
range $=[-3,\infty)$



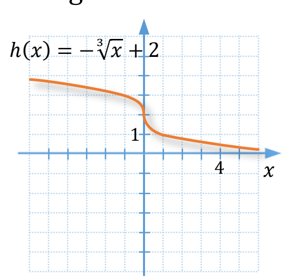
Translation: 3 steps down

61. $D=\mathbb{R}$
range $=\mathbb{R}$



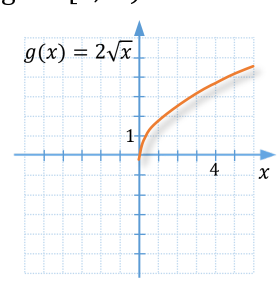
Translation: 2 steps to the right

63. $D=\mathbb{R}$
range $=\mathbb{R}$

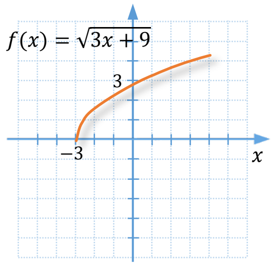


Reflection in x-axis,
Translation 2 steps up

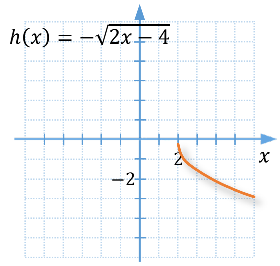
65. $D=[0,\infty)$
range $=[0,\infty)$



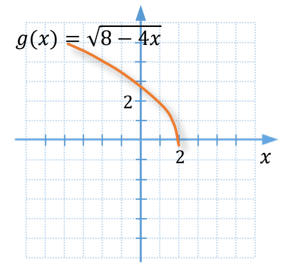
67. $D = [-3, \infty)$
range $= [0, \infty)$



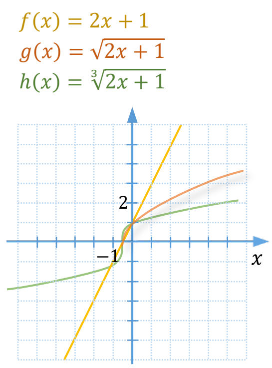
69. $D = [2, \infty)$
range $= (-\infty, 0]$



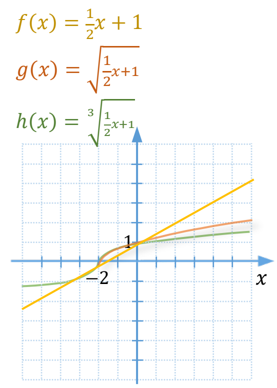
71. $D = (-\infty, 2]$
range $= [0, \infty)$



73.



75.



77. ~186 cm

79. 2 sec

81. approx. 2711 m^2

RD2 Exercises

1. **a.-B.; b.-A.; c.-C.; d.-F.; e.-D.; f.-E.**

3. 2

5. -343

7. $-\frac{1}{10}$

9. $\frac{8}{27}$

11. not a real number

13. -2

15. $5^{\frac{1}{2}}$

17. $x^{\frac{6}{2}} = x^3$

19. $64^{\frac{1}{3}}x^{\frac{6}{3}} = 4x^2$

21. $\frac{25^{\frac{1}{2}}}{x^{\frac{5}{2}}} = \frac{5}{x^{\frac{5}{2}}}$

23. $\left(\sqrt{4}\right)^5 = 32$

25. $\sqrt[5]{x^3}$

27. $\sqrt[3]{(-3)^2} = \sqrt[3]{9}$

29. $\frac{2}{\sqrt{x}}$

31. $3^{\frac{7}{8}}$

33. $2^{\frac{3}{4}}$

35. $5^{\frac{5}{4}}$

37. $x^{\frac{1}{2}} \cdot y^{\frac{10}{3}}$

39. $\frac{x^{\frac{5}{9}}}{y^{\frac{1}{2}}}$

41. $5x^{\frac{4}{15}}$

43. $\sqrt[3]{x}$

45. y^{-3} or $\frac{1}{y^3}$

47. $\sqrt[3]{9}$ **49.** $2y^2$ **51.** $2x^2\sqrt[3]{2y^2}$ **53.** $2x\sqrt{y}$

55. $\sqrt[6]{5^5}$ **57.** $\sqrt[6]{9a^5}$ **59.** $x\sqrt{x}$ **61.** $\frac{\sqrt{x}}{x^2}$ or $\frac{1}{x\sqrt{x}}$

63. $\frac{2}{\sqrt[12]{x^5}}$ **65.** $\sqrt[12]{xy}$ **67.** $\sqrt[24]{x}$ **69.** $\sqrt[8]{x^3}$

71. To treat an equation as an identity, the equation must be true for <u>all</u> variable values in the domain. The fact that the equation is true for specific values does not guarantee that it is true for all values of x and y. A counterexample: Let $x = y = 2$. Then $\sqrt[n]{2^n + 2^n} = \sqrt[n]{2 \cdot 2^n} = 2\sqrt[n]{2} \neq 2 + 2 = 4$.

73. 30 beats per minute

RD3 Exercises

1. 5 **3.** $3\sqrt{2}$ **5.** $30\sqrt{3}$ **7.** $3x^4\sqrt{2}$

9. $4x^3y\sqrt{6xy}$ **11.** $2x^2$ **13.** $3\sqrt{2}$ **15.** $\sqrt{6}$

17. $2b\sqrt{b}$ **19.** $4x\sqrt{y}$ **21.** 2 **23.** $2a\sqrt[3]{b}$

25. $12x^2y^4\sqrt{y}$ **27.** $-5a^2b^3c^4$ **29.** $\frac{m^2n^5}{2}$ **31.** $a^3b^3\sqrt{7a}$

33. $2x^2y^3\sqrt[5]{2x^2}$ **35.** $-3a^3b^2\sqrt[4]{2a^3b^2}$ **37.** $\frac{4}{7}$ **39.** $\frac{11}{y}$

41. $\frac{3a\sqrt[3]{a^2}}{4}$ **43.** $\frac{2x^3}{yz^4}$ **45.** $\sqrt{6}$ **47.** $-x^2\sqrt{x}$

49. $-\frac{1}{x\sqrt{xy}}$ **51.** $\frac{x^2\sqrt[6]{x}}{yz^2}$

53. This is not correct as the radical of a sum is <u>not</u> the sum of radicals. We can simplify it by factoring the radicand: $\sqrt{x^3 + x^2} = \sqrt{x^2(x+1)} = |x|\sqrt{x+1}$

55. $\sqrt[10]{x^7}$ **57.** $2\sqrt[15]{2^4}$ or $2\sqrt[15]{16}$ **59.** $\sqrt[4]{x}$ **61.** $\sqrt[15]{\frac{2^7}{a^4}}$

63. $\sqrt[6]{2x^5}$ **65.** $\sqrt[12]{x^{11}}$ **67.** $\sqrt{6}$ **69.** $\sqrt{n^2 - 9}$

71. $2\sqrt{31}$ **73.** $2\sqrt{5}$ **75.** $\frac{\sqrt{41}}{7}$ **77.** $2\sqrt{38}$

79. $\sqrt{p^2 + q^2}$ **81.** ~7.05 meters **83.** $(-4, 0)$ and $(4, 0)$ **85.** 30 m

RD4 Exercises

1. No. The equation must be true for all $x \geq 0$.
3. $7\sqrt{3}$
5. $13y\sqrt{3x}$
7. $14\sqrt{2} + 2\sqrt{3}$
9. $11\sqrt[3]{2}$
11. $(1 + 6a)\sqrt{5a}$
13. $(4x - 6)\sqrt{x}$ or $2(2x - 3)\sqrt{x}$
15. $24\sqrt{2x}$
17. $(x + 1)\sqrt[3]{6x}$
19. $-8n\sqrt{2}$
21. $(6ab^2 - 9ab)\sqrt{ab}$ or $3ab(2b - 3)\sqrt{ab}$
23. $5x\sqrt[4]{xy}$
25. $x\sqrt[3]{2x} + \sqrt{2}$
27. $\sqrt{x + 3}$
29 $(5 - x)\sqrt{x - 1}$
31. $\frac{3\sqrt{3}}{4}$
33. $-\frac{2a\sqrt[4]{a}}{3}$
35. Error: cannot add unlike radicals (see line 3). Correct solution: $2\sqrt{2} + 2\sqrt[3]{2} = 2\left(\sqrt{2} + \sqrt[3]{2}\right)$
37. $3\sqrt{5} - 10$
39. $10 - 2\sqrt{5}$
41. -6
43. 1
45. -13
47. $30 - 10\sqrt{5}$
49. $a - 25b$
51. $9 + 6\sqrt{2}$
53. $38 + 12\sqrt{10}$
55. $22 - 13\sqrt{3}$
57. $\sqrt[3]{4y^2} - 4\sqrt[3]{2y} - 5$
59. 1
61. $(f + g)(x) = 13x\sqrt{5x}$; $(fg)(x) = 150x^3$
63. $\frac{\sqrt{10}}{4}$
65. $2\sqrt{6}$
67. $-\sqrt{5}$
69. $\frac{\sqrt{10y}}{8}$
71. $\frac{y\sqrt[3]{9x^2y}}{3x^2}$
73. $\sqrt[4]{pq^3}$
75. $\frac{6-\sqrt{2}}{2}$
77. $6 + 2\sqrt{6}$
79. $\frac{3\sqrt{5}-2\sqrt{3}}{11}$
81. $\sqrt{m} - 2$
83. $\frac{3+4\sqrt{3x}+4x}{3-4x}$
85. $\frac{2a+2\sqrt{ab}}{a-b}$
87. $1 - 2\sqrt{5}$
89 $\frac{2-9\sqrt{2}}{3}$
91. $\frac{6-2\sqrt{6p}}{3}$
93. Yes. $\frac{\sqrt{3}-1}{1+\sqrt{3}}$ after rationalization of the denominator becomes $2 - \sqrt{3}$.
95. $2\sqrt{3} \approx 3.5$ cm

RD5 Exercises

1. False, as the radicals do not contain a variable.
3. True, as the radical cannot equal a negative.
5. $x = \frac{39}{7}$
7. $x = \frac{2}{3}$
9. no solution
11. $x = -27$

13. $y = 19$ **15.** $a = \frac{1}{25}$ **17.** $r = 5$ **19.** $y = 18$

21. $x = 9$ **23.** $x \in \{-1, 3\}$ **25.** $y = 4$ **27.** $x = 5$

29. not correct, as $(4 - x)^2 = 16 - 8x + x^2$ **31.** $x = 2$ **33.** $p = 9$

35. No solution **37.** $t = -1$ **39.** No solution **41.** $n = 3$

43. $n = -2$ **45.** $a \in \{2, 6\}$ **47.** No solution **49.** $m = 2$

51. $x \in \left\{-1, \frac{1}{3}\right\}$ **53.** $x \in \{1, 9\}$ **55.** $x = \frac{4}{9}$ **57.** $k \in \{-2, -1\}$

59. $x \in \{-5, 5\}$ **61.** $a \in \left\{0, \frac{125}{4}\right\}$ **63.** $L = CZ^2$ **65.** $m = \frac{2K}{V^2}$

67. $F = \frac{Mm}{r^2}$ **69.** $C = \frac{1}{4\pi^2 F^2 L}$ **71.** $r = \frac{a}{4\pi^2 N^2}$ **73.** 189 cm

75. 22 m

RD6 Exercises

1. The mistake is in the first step – the product rule for radicals cannot be used, so we need to convert into i notation before multiplying: $\sqrt{-3} \cdot \sqrt{-15} = \sqrt{3}\, i \cdot \sqrt{15}\, i = \sqrt{45}\, i^2 = 3\sqrt{5}\,(-1) = \mathbf{-3\sqrt{5}}$

3. Both are correct because $(8i)^2 = 8^2 \cdot -1 = -64$ and $(-8i)^2 = (-8)^2 \cdot -1 = -64$.

5. $10i$ **7.** $7\sqrt{2}\, i$ **9.** -7 **11.** $-7\sqrt{3}$

13. $-4\sqrt{2}\, i$ **15.** $24\sqrt{10}\, i$ **17.** $-73 + 31i$ **19.** $-90 - 46\sqrt{3}\, i$

21. 112 **23.** $18 + 2i$ **25.** $6 - 82i$ **27.** $-13 + 84i$

29. 181 **31.** 1 **33.** i **35.** $-i$

37. $2 - 2\sqrt{14}\, i$ **39.** $\frac{1}{5} + \frac{\sqrt{39}}{10} i$ **41.** $-\frac{6}{5} i$ **43.** $\frac{15}{74} - \frac{21}{74} i$

45. $\frac{7}{25} + \frac{24}{25} i$ **47.** $\frac{97}{137} + \frac{27}{137} i$ **49.** Yes, because $(-2i)^2 = (-2)^2 i^2 = -4$.

51. Yes, because substituting $x = 3 - 2i$ into the equation gives $(3 - 2i)^2 - 6(3 - 2i) + 13 = (9 - 12i + 4i^2) - 18 + 12i + 13 = 4 + 4i^2 = 4 - 4 = 0$.

53. No, because substituting $x = 5 + i$ into the equation gives $(5 + i)^2 + 5(5 + i) + 60 = (25 + 10i + i^2) + 25 + 5i + 60 = 110 + 15i + i^2 = 110 + 15i - 1 = 109 + 15i \neq 0$.