Quadratic Equations and Functions - ANSWERS

Q1 Exercises

1. False

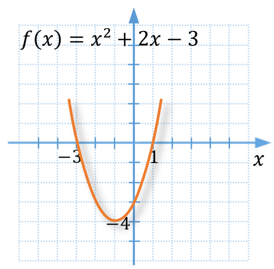
3. True

5. False

7. False

9. **a)**

x	$f(x)$
1	0
0	-3
−1	-4
−2	-3
−3	0



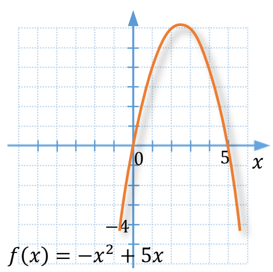
b) $(-3,0)$, $(1,0)$

c) $x \in \{-3,1\}$

The solutions are the first coordinates of the x-intercepts.

11. **a)**

x	$f(x)$
0	0
1	4
2	6
$\frac{5}{2}$	6.25
3	6
4	4
5	0



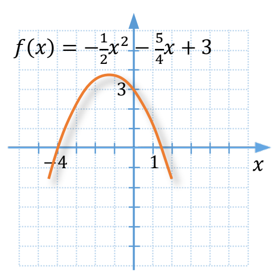
b) $(0,0)$, $(5,0)$

c) $x \in \{0,5\}$

The solutions are the first coordinates of the x-intercepts.

13. **a)**

x	$f(x)$
−4	0
−2	3.5
−1	3.75
0	3
1	1.25
2	-1.5



b) $(-4,0)$, $\left(\frac{3}{2},0\right)$

c) $x \in \left\{-4, \frac{3}{2}\right\}$

The solutions are the first coordinates of the x-intercepts.

15. $x \in \{-4\sqrt{2}, 4\sqrt{2}\}$

17. $n \in \{-2\sqrt{6}, 2\sqrt{6}\}$

19. $y \in \{-2\sqrt{10}, 2\sqrt{10}\}$

21. $x \in \{-7,1\}$

23. $t \in \left\{\frac{-2-2\sqrt{3}}{5}, \frac{-2+2\sqrt{3}}{5}\right\}$

25. $y \in \{-4-2\sqrt{11}, -4+2\sqrt{11}\}$

27. $y \in \left\{\frac{44}{5}, \frac{56}{5}\right\}$

29. $x \in \left\{\frac{-3-5i}{4}, \frac{-3+5i}{4}\right\}$

31. $x \in \left\{\frac{1-3\sqrt{2}}{2}, \frac{1+3\sqrt{2}}{2}\right\}$

33. $y \in \{0,3\}$

35. $n = -2$

37. $y \in \left\{\frac{-7-\sqrt{53}}{2}, \frac{-7+\sqrt{53}}{2}\right\}$

39. $a \in \{-1-\sqrt{2}\,i, -1+\sqrt{2}\,i\}$

41. $x \in \{6 - 2\sqrt{5}, 6 + 2\sqrt{5}\}$

43. $x \in \left\{\frac{-1-\sqrt{7}}{3}, \frac{-1+\sqrt{7}}{3}\right\}$

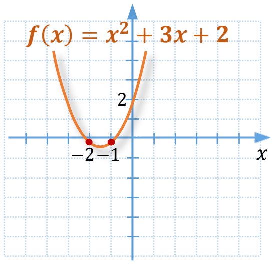
45. $x \in \left\{\frac{4-\sqrt{3}}{3}, \frac{4+\sqrt{3}}{3}\right\}$

47. $x \in \left\{\frac{2-\sqrt{3}}{3}, \frac{2+\sqrt{3}}{3}\right\}$

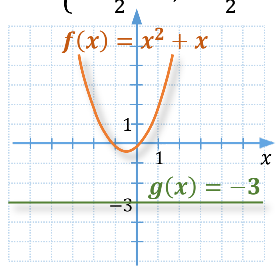
49. $x \in \left\{\frac{1-2\sqrt{19}}{5}, \frac{1+2\sqrt{19}}{5}\right\}$

51. $x \in \{1 - 2\sqrt{2}, 1 + 2\sqrt{2}\}$

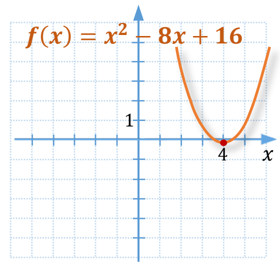
53. $x \in \{-2, -1\}$



55. $x \in \left\{\frac{-1-\sqrt{11}\,i}{2}, \frac{-1+\sqrt{11}\,i}{2}\right\}$



57. $x = 4$



59. $a = 1 - \sqrt{5} \approx -1.24$, or $a = 1 + \sqrt{5} \approx 3.24$

61. $x = \frac{-5-\sqrt{11}}{2} \approx -4.16$, or $x = \frac{-5+\sqrt{11}}{2} \approx -0.84$

63. $y = \frac{-1-\sqrt{7}}{6} \approx -0.61$, or $y = \frac{-1+\sqrt{7}}{6} \approx 0.27$

65. $x = \frac{17-\sqrt{249}}{10} \approx 0.12$, or $x = \frac{17+\sqrt{249}}{10} \approx 3.28$

67. $x = \frac{5-\sqrt{7}}{6} \approx 0.39$, or $x = \frac{5+\sqrt{7}}{6} \approx 1.27$

69. 2 rational solutions; factoring possible

71. 1 double rational solution; factoring possible

73. 1 double rational solution; factoring possible

75. $k = 25$

77. No, as the product of a rational and irrational number is irrational. This would contradict the fact that the quadratic equation has integral coefficients.

79. $x = 1 \pm \sqrt{10}$

81. $x = \frac{5 \pm 2\sqrt{6}}{2}$

83. $x \in \{-3, 2\}$

85. $x = -1 \pm 2i$

87. $x \in \left\{-\frac{3}{2}, 1\right\}$

89. $x = 5 \pm \sqrt{53}$

Q2 Exercises

1. The solution is incorrect as the question calls for the values of x not a.

3. $x \in \{\pm\sqrt{5}, \pm\sqrt{2}, \pm i\sqrt{2}, \pm i\sqrt{5}\}$

5. $x \in \left\{\frac{1}{4}, 16\right\}$

7. $a \in \{-1, 2\}$

9. $x = 9$

11. $x \in \{-1, 3, 1 - \sqrt{2}, 1 + \sqrt{2}\}$

13. $x = 8$

15. $u \in \left\{-\frac{8}{3}, -1\right\}$

17. $x \in \{-1 \pm \sqrt{2}, 3 \pm \sqrt{10}\}$

19. $r = \pm\sqrt{\frac{V}{\pi h}}$

21. $s = \pm\sqrt{\frac{3V}{h}}$

23. $s = \pm\sqrt{\frac{kq_1q_2}{N}}$

25. $H = \pm\sqrt{\frac{703W}{I}}$

27. $r = \frac{-\pi h \pm \sqrt{\pi^2 h^2 + 2\pi A}}{2\pi}$

29. $a = \pm\frac{bt}{\sqrt{1-t^2}}$

31. $I = \frac{E\pm\sqrt{E^2-4PR}}{2R}$

33. $v = \pm\frac{c\sqrt{m^2-m_0^2}}{m}$

35. $\pm\frac{(r+R)\sqrt{pR}}{R}$

37. **a.** $r - c$ **b.** $r + c$

39. $7 + 2\sqrt{35},\ 10 + 2\sqrt{35},$ and $17 + 2\sqrt{35}$

41. 5 ft by 12 ft

43. 10 ft 3 in

45. 9 in by 13 in

47. $10\sqrt{2}$ m by $5\sqrt{2}$ m

49. 1.5 ft

51. 12 cm

53. 7 cm by 13 cm

55. 60 km/h

57. Skyhawk at 250 km/h; Mooney Bravo at 350 km/h

59. ~10.8 km/h

61. 800 km/h and 840 km/h

63. 7 hr 19 min

65. Helen: ~16 hr 31 min; Monica: ~15 hr 31 min

67. smaller-diameter pipe: 3 hr;
larger-diameter pipe: 2 hr

69. ~3.8 sec

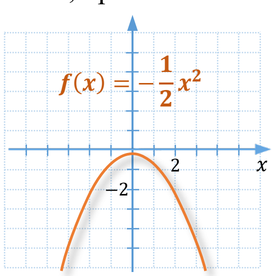
71. 4.2%

Q3 Exercises

1. a.-III; b.-I; c.-IV; d.-II

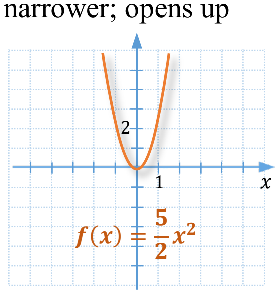
3. a.-II; b.-III; c.-I; d.-IV

5. wider; opens down



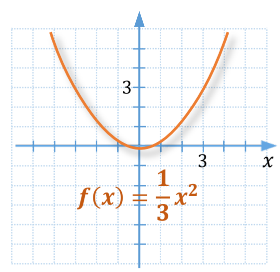
Range $= (-\infty, 0]$

7. narrower; opens up



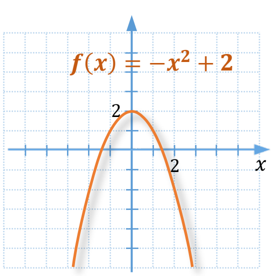
Range $= [0, \infty)$

9. wider; opens up



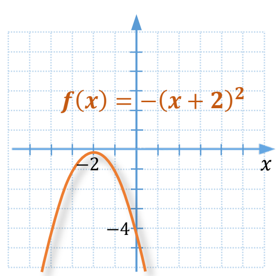
Range $= [0, \infty)$

11. flip; shift 2 units up



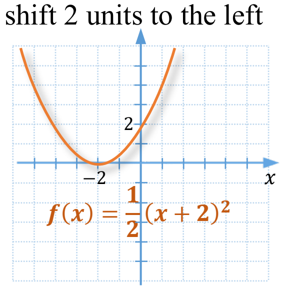
Domain = ℝ
Range $= (-\infty, 2]$
Axis of symmetry: $x = 0$

13. flip; shift 2 units left



Domain = ℝ
Range $= (-\infty, 0]$
Axis of symmetry: $x = -2$

15. vertical dilation by $\frac{1}{2}$;
shift 2 units to the left



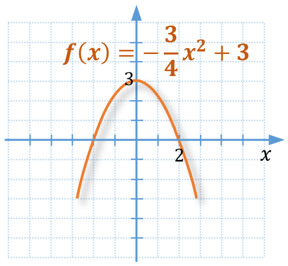
Domain = ℝ
Range $= [0, \infty)$
Axis of symmetry: $x = -2$

17. vertex = $(0,3)$

shape of $\frac{3}{4}x^2$; opens down

x-int.: $(-2,0)$, $(2,0)$

y-int.: $(0,3)$



flip; vertical contraction by $\frac{3}{4}$;

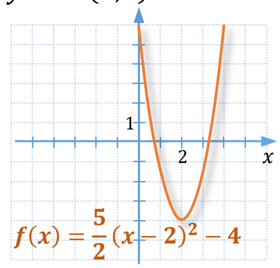
shift 3 units up

19. vertex = $(2,-4)$

shape of $\frac{5}{2}x^2$; opens up

x-int.: $\left(\frac{10-2\sqrt{10}}{5}, 0\right)$, $\left(\frac{10+2\sqrt{10}}{5}, 0\right)$

y-int.: $(0,6)$



vertical dilation by $\frac{5}{2}$;

shift 2 units to the right;

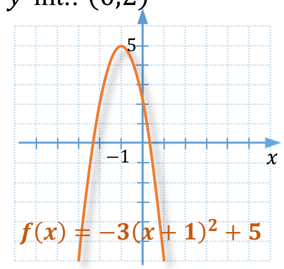
shift 4 units down

21. vertex = $(-1,5)$

shape of $3x^2$; opens down

x-int.: $\left(\frac{-3-\sqrt{15}}{3}, 0\right)$, $\left(\frac{-3+\sqrt{15}}{3}, 0\right)$

y-int.: $(0,2)$



flip; vertical dilation by 3;

shift 1 unit to the left;

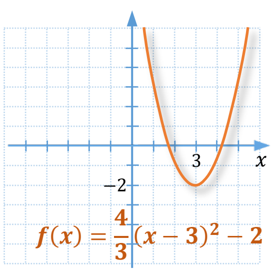
shift 5 units up

23. vertex = $(3,-2)$

shape of $\frac{4}{3}x^2$; opens up

x-int.: $\left(\frac{6-\sqrt{6}}{2}, 0\right)$, $\left(\frac{6+\sqrt{6}}{2}, 0\right)$

y-int.: $(0,10)$

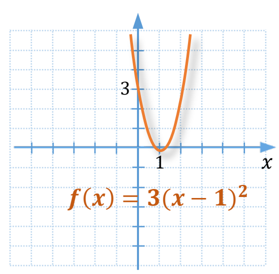


vertical dilation by $\frac{4}{3}$;

shift 3 units to the right;

shift 2 units down

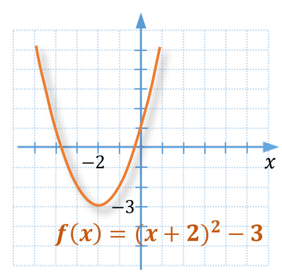
25. vertex = $(1,0)$



Minimum value $= 0$;

Range $= [0, \infty)$

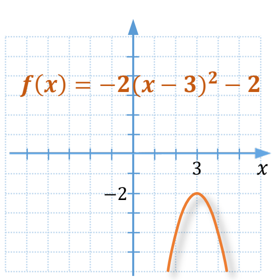
27. vertex = $(-2,-3)$



Minimum value $= -3$;

Range $= [-3, \infty)$

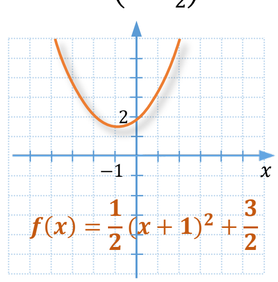
29. vertex = $(3,-2)$



Maximum value $= -2$;

Range $= (-\infty, -2]$

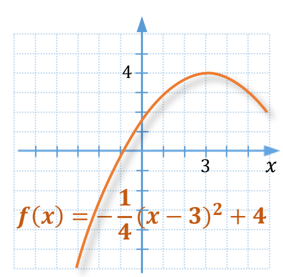
31. vertex = $\left(-1, \frac{3}{2}\right)$



Minimum value $= \frac{3}{2}$;

Range $= \left[\frac{3}{2}, \infty\right)$

33. vertex = $(3,4)$



Maximum value $= 4$;

Range $= (-\infty, 4]$

35. $f(x) = (x+3)^2 - 4$

37. $f(x) = 2(x-1)^2 - 5$

39. $f(x) = -3(x+2)^2 + 6$

Q4 Exercises

1. $f(x) = (x+3)^2 + 1;\ V(-3,1)$

3. $f(x) = \left(x+\frac{1}{2}\right)^2 - \frac{13}{4};\ V(-\frac{1}{2}, -\frac{13}{4})$

5. $f(x) = -\left(x-\frac{7}{2}\right)^2 + \frac{61}{4};\ V\left(\frac{7}{2}, \frac{61}{4}\right)$

7. $f(x) = -3(x-1)^2 + 15;\ V(1,15)$

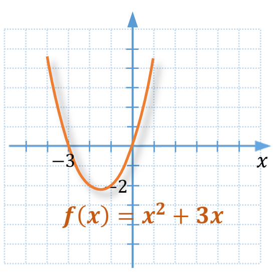
9. $f(x) = \frac{1}{2}(x+3)^2 - \frac{11}{2};\ V\left(-3, -\frac{11}{2}\right)$

11. $V\left(\frac{3}{2}, -\frac{11}{4}\right)$

13. $V(1,8)$

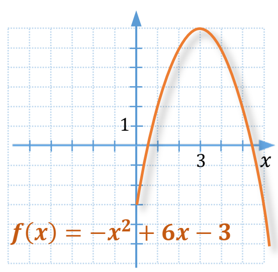
15. $V(-1,-23)$

17. $V\left(-\frac{3}{2}, -\frac{9}{4}\right)$; opens up
shape of x^2



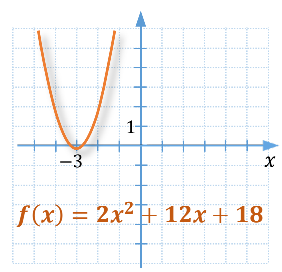
$D = \mathbb{R}$; Range $= \left[-\frac{9}{4}, \infty\right)$

19. $V(3,6)$; opens down
shape of x^2



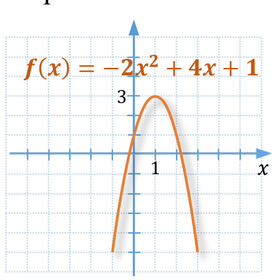
$D = \mathbb{R}$; Range $= (-\infty, 6]$

21. $V(-3,0)$; opens up
shape of $2x^2$



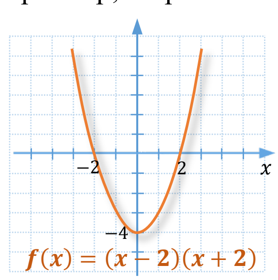
$D = \mathbb{R}$; Range $= [0, \infty)$

23. $V(1,3)$; opens down
shape of $2x^2$



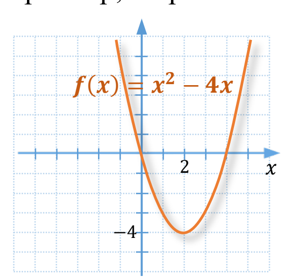
$D = \mathbb{R}$; Range $= (-\infty, 3]$

25. zeros: -2, 2; $V(0,-4)$;
opens up; shape of x^2



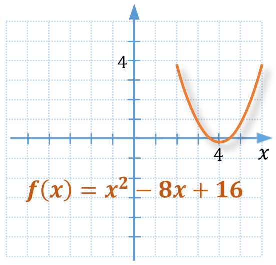
Minimum value $= -4$
Minimum occurs at $x = 0$

27. zeros: 0, 4; $V(2,-4)$;
opens up; shape of x^2



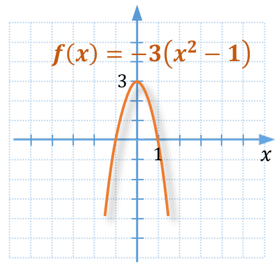
Minimum value $= -4$
Minimum occurs at $x = 2$

29. zero: 4; $V(4,0)$;
opens up; shape of x^2



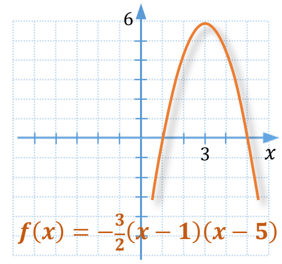
Minimum value $= 0$
Minimum occurs at $x = 4$

31. zeros: -1, 1; $V(0,3)$;
opens down; shape of $3x^2$



Maximum value $= 3$
Maximum occurs at $x = 0$

33. zeros: 1, 5; $V(3,6)$;
opens down; shape of $\frac{3}{2}x^2$



Maximum value $= 6$
Maximum occurs at $x = 3$

35. $f(x) = x(5x - 2)$

37. $f(x) = \frac{3}{4}(x - 1)(x - 4)$

39. $x(3x - 1) = 0$

41. $(x - 2)^2 = 0$

43. By observing the second coordinate of the vertex in combination with the opening. For example, the second coordinate "+'ve" with opening up means no x-intercepts while the second coordinate "+'ve" with opening down indicates 2 x-intercepts.

45. true

47. true

49. true

51. 30.625 m; 5 sec

53. 20; \$150

55. 16, 16

57. 4 m by 8 m

59. **a.** $P(n) = 60 - 2n$ **b.** $R(n) = (60 - 2n)n$ **c.** 15 **d.** 450\$