## Unit 1

## Common Fractions

## Topic A: Introducing Common Fractions

This unit gives you the background details that you need for working with common fractions.

Common fractions are written with two numbers, one above the other, with either a straight or slanted line in between. The straight line style is the one used most.


The denominator is the bottom number. It tells how many equal parts are in the whole thing. The numerator is the top number. It tells how many of the equal parts we are dealing with.


The whole thing is the bunch of bananas.
The whole thing has 5 equal parts (the bananas). The denominator is 5 .

How many bananas have been eaten? 1
What fraction of the bananas have been eaten?
$\frac{1}{5}$ of the bananas

The whole thing is a carton of one dozen eggs. One dozen has 12 equal parts, so if we are talking about the carton, the denominator is $\mathbf{1 2}$. How many eggs are still in the carton? 7
What fraction of the eggs are left?
$\frac{7}{12}$ of the eggs

A fraction is always looking at things as parts of a whole. In the example of the eggs above, the whole is 12 eggs. The part is the 7 eggs that are left. 7 is part of the whole of 12 .

## Example A:

This pizza is one whole pizza. The pizza is cut into 8 pieces. This means the whole is 8 . How many parts are left? (the pieces that are shaded are the ones left) Write a fraction of how many pieces of pizza are left:


The amount left over can be shown as a fraction: $\frac{5}{8}$

## Example B:

It is Peter's $82^{\text {nd }}$ birthday. There were 8 people, including Peter, at the party. Everyone wants a small piece of cake, so Kathleen cut the cake into 12 equal parts. This means the whole is 12. There will be some left over.

The amount left over can be shown as a fraction: $\frac{4}{12}$


## Example C:

Sue made a strawberry pie to share with her family of 4 . The pie was cut into 8 equal parts. This means the whole is 8 . The kids are excited because there will be parts left over.

The fraction showing what amount of pie is left is: $\frac{4}{8}$
 terms you are given:

Numerator
Denominator
Whole
Fraction


## Answers to Exercise One



## Writing Common Fractions

Exercise Two Each shape drawn here is a whole. The shapes have been divided into parts.

1. Ask yourself "How many equal parts in the whole?" That number is the denominator.
2. Count the number of parts that are shaded $\square$; that is the numerator.
3. Write the common fraction to describe the shaded portion of each shape.

a) i) How many parts make the whole? 4
b) i) How many parts make the whole? $\qquad$
ii) How many parts are shaded? $\qquad$ iii) Fraction: $\frac{3}{4}$
ii) How many parts are shaded? $\qquad$
iii) Fraction: $\qquad$

c) i) How many parts make the whole?
ii) How many parts are shaded? $\qquad$ iii) Fraction:

e) i) How many parts make the whole? $\qquad$
ii) How many parts
are shaded? $\qquad$
iii) Fraction: $\qquad$

d) i) How many parts make the whole? $\qquad$
ii) How many parts are shaded? $\qquad$
iii) Fraction: $\qquad$

f) i) How many parts make the whole?
ii) How many parts are shaded? $\qquad$
iii) Fraction: $\qquad$

g) i) How many parts make the whole? $\qquad$
ii) How many parts are shaded? $\qquad$
iii) Fraction: $\qquad$

i) i) How many parts make the whole? $\qquad$
ii) How many parts are shaded? $\qquad$
iii) Fraction:

h) i) How many parts make the whole? $\qquad$
ii) How many parts are shaded? $\qquad$
iii) Fraction: $\qquad$

j) i) How many parts make the whole?
ii) How many parts are shaded? $\qquad$
iii) Fraction: $\qquad$

## Answers to Exercise Two

$\begin{array}{lll}\text { a) i) } 4 & \text { ii) } 3 & \text { iii) } \frac{3}{4}\end{array}$
b) i) 8 ii) 5 iii) $\frac{5}{8}$
c) i) 5 ii) 2 iii) $\frac{2}{5}$
$\begin{array}{lll}\text { d) i) } 4 & \text { ii) } 1 & \text { iii) } \frac{1}{4}\end{array}$
f) i) 13 ii) $5 \quad$ iii) $\frac{5}{13}$
g) i) 4 ii) 2 iii) $\frac{2}{4}$
h) i) 3 ii) $1 \quad$ iii) $\frac{1}{3}$
i) i) 6
ii) 5 iii) $\frac{5}{6}$
$\begin{array}{lll}\text { j) i) } 9 & \text { ii) } 4 & \text { iii) } \frac{4}{9}\end{array}$

Exercise Three
Now draw some fractions.

Example:
Draw the fraction $\frac{1}{2}$ in a circle:

a) draw $\frac{1}{4}$ in a circle:
b) draw $\frac{1}{3}$ in a circle (here is a hint how):

c) draw $\frac{1}{2}$ in the rectangle:
$\square$
d) draw $\frac{2}{4}$ in the rectangle:

e) draw $\frac{4}{8}$ in the rectangle:

f) What do you see in common with the three last boxes you just drew?
$\qquad$
$\qquad$
$\qquad$

## Answers to Exercise Three

a)

b)

c)

d)

e)

f) all the shaded spaces are the same.

## How do we read common fractions?

You can read fractions in a few different ways:

| $\frac{1}{2}$ can be called: | One over two <br> One half |
| :--- | :--- |
| $\frac{1}{4}$ can be called: | One over four <br> One fourth |
|  | One quarter |
| $\frac{3}{4}$ can be called: | Three over four <br> Three fourths |
|  | Three quarters |
| $\frac{1}{3}$ can be called: | One over three <br> One third |


| If the <br> denominator is... | Read... |
| :---: | :--- |
| 2 | half |
| 3 | third |
| 4 | fourth or quarter |
| 5 | fifth |
| 6 | sixth |
| 7 | seventh |
| 8 | eighth |
| 9 | ninth |
| 10 | tenth |
| 22 | twenty-second |

## (these are called ordinal numbers)

Add the "s" if the numerator is 2 or more.
So $\frac{2}{3}$ is read "two-thirds".
Note that the usual practice is to put a hyphen (-) between the words when you write them out.
$\frac{1}{2}$ is usually read "one-half". $\frac{2}{2}$ is read "two-halves"
$\frac{3}{4}$ is read as "three-quarters" or "three-fourths".

## Exercise Four

Look back again at Exercise Two and write down the word names for your answers.
a) three-quarters or three-fourths
b) $\qquad$
c) $\qquad$
d) $\qquad$
e)
f) $\qquad$
g) $\qquad$
h)
i) $\qquad$
j) $\qquad$

## Answers to Exercise Four

a) three quarters or three fourths
b) five eighths
c) two fifths
d) one quarter or one fourth
e) one half
f) five thirteenths
g) two quarters or two fourths
h) one third
i) five sixths
j) four ninths

We make common fractions out of many things in our lives. For example,

- I got 13 out of 15 on my English test. The score is $\frac{13}{15}$.
- The baseball pitcher struck out 2 of the 6 batters in the inning.
$\frac{2}{6}$ of the batters were struck out.
- Three of the eggs in that dozen are cracked. $\frac{3}{12}$ of the eggs are cracked.
- Finish your vegetables. I gave you just 8 pieces of carrot, and you have only eaten 4 of them! $\frac{4}{8}$ of the carrots are eaten.


## Exercise Five

Answer the questions using a common fraction.
a) $\frac{20}{30}$ Jill walks for 20 minutes of the 30 minute lunch break. What fraction of her lunch break does Jill walk?
b) $\qquad$ The test was scored out of 25 . Kim got 20 marks. Write his score.
c) $\qquad$ The restaurant has 12 tables. Each waiter looks after 6 of them. What fraction of the tables does each waiter look after?
d) $\qquad$ The new litter of puppies is a big one-10 pups. Three of the pups have floppy ears. What fraction of the puppies have floppy ears?
e) $\qquad$ Beryl planted 3 dozen tulip bulbs last fall. A mole ate one dozen of them before they flowered. That mole is in trouble!! What fraction of the tulips did the mole eat?
f) ___ Kay's raisin cookie recipe uses 5 cups of flour altogether. Kay always puts in 2 cups of whole wheat flour and 3 cups of white flour. What fraction of the flour that she uses is whole wheat?
g) $\qquad$ Greg got 6 new golf balls for his birthday. On his first golf game, he lost two of them in the water trap. What fraction of the new balls has he lost?
h) $\qquad$ The class has 18 students. 16 of the students are enrolled in the math course. What fraction of the class is taking math?
i) $\qquad$ Kay made 72 delicious raisin cookies last night. Her teenage sons and their friends ate 36 of them. What fraction of the cookies did they eat?
j) $\qquad$ Dave bought four litres of oil for his car. But when he changed the oil, he only needed to put in three litres. What fraction of the oil did he use?

Answers to Exercise Five
a) $\frac{20}{30}$
b) $\frac{20}{25}$
c) $\frac{6}{12}$
d) $\frac{3}{10}$
e) $\frac{1}{3}$
f) $\frac{2}{5}$
g) $\frac{2}{6}$
h) $\frac{16}{18}$
i) $\frac{36}{72}$
j) $\frac{3}{4}$

These common fractions that you have been writing are called proper fractions.
Proper fractions are fractions were the numerator is smaller than the denominator.

## How do we compare common fractions?

## Example A:

a) Which circle has more shaded parts?

b) Yes, the one on the right has more shaded parts.
c) Which is larger: $\frac{1}{4}$ or $\frac{2}{4}$ ?
(Look back at the circles above to help answer this question).

Answer: $\frac{2}{4}$ is larger because it fills in more parts of the circle.

## Example B:

Review: Greater than $\quad \gg$ Less than $\quad L^{\circ}$ Greater than


a) Which is shaded more?
b) Write the fractions for both drawings:
$\qquad$ or

c) Which fraction is larger? Place a symbol ( $<$ or $>$ ) in the box above to show your answer.

Answer: $\frac{1}{4}<\frac{3}{4}$ is correct!

## Exercise Six

1) 


a) Shade

$\frac{1}{6}$ and $\frac{5}{6}$
b) Circle the fraction that is larger.
c) Write a mathematical sentence stating which fraction is larger (use $<$ or $>$ )

2)

$\frac{4}{6}$
a) Shade:
and

b) Circle the fraction that is larger.
c) Write the mathematical sentence stating which fraction is larger (Use $<$ or $>$ ).

3)

a) Shade:
$\frac{2}{6}$
and
b) Circle the fraction that is larger.
c) Write a mathematical sentence that shows which fraction is larger.
$\qquad$

$\qquad$

Answers to Exercise Six

1) $\frac{1}{6}<\frac{5}{6}$
2) $\frac{4}{6}>\frac{3}{6}$
3) $\frac{2}{6}>\frac{1}{6}$

There is a rule you can follow to compare fractions:

As the numerator gets larger and the denominator stays the same, the fraction gets larger.

Example: $\frac{3}{10}<\frac{7}{10}$

Exercise Seven
a) $\frac{3}{4} \quad \frac{1}{4}$
d) $\frac{9}{10}$
$\xrightarrow{\frac{1}{10}}$
b) $\frac{5}{6}$ $\xrightarrow{\frac{1}{6}}$
$\frac{1}{6}$
e) $\frac{3}{8}$
$\square$
c) $\frac{3}{5}$ $\xrightarrow{\square}$
$\frac{4}{5}$
f) $\frac{1}{5}$
$\qquad$
$\frac{4}{5}$

## Answers to Exercise Seven

a) $>$
b) $>$
c) $<$
d) $>$
e) <
f) $<$
a) One fourth $\leq$ Three fourths
d) Five ninths $\qquad$ two ninths
b) Five sixths $\qquad$ Four sixths
e) Seven sevenths $\qquad$ three sevenths
c) One eighth $\qquad$ three eighths
f) One third $\qquad$ two thirds
Answers to Exercise Eight
b) $>$
c) $<$
d) $>$
e) $>$
f) $<$

Exercise Nine
a) $\frac{3}{4}, \frac{1}{4}, \frac{2}{4}$
b) $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}, \frac{1}{7}, \frac{5}{7}, \frac{4}{7}$
c) $\frac{4}{10}, \frac{7}{10}, \frac{1}{10}, \frac{5}{10}, \frac{9}{10}$
d) $\frac{50}{361}, \frac{23}{361}, \frac{7}{361}, \frac{360}{361}, \frac{274}{361}, \frac{158}{361}$

## Answers to Exercise Nine

a) $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$
b) $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$
c) $\frac{1}{10}, \frac{4}{10}, \frac{5}{10}, \frac{7}{10}, \frac{9}{10}$
d) $\frac{7}{361}, \frac{23}{361}, \frac{50}{361}, \frac{158}{361}, \frac{274}{361}, \frac{360}{361}$

Using a number line is another way to look at how numbers compare to each other. Fractions can also be plotted on a number line. The number line is numbered 0 to 2 . The section between 0 and 1 is split into fractions.

Draw a line to connect the fractions listed to the fractions on the number line.

## Example:



Exercise Ten Draw a line between the following fractions and the fractions on the number line.
a)

b)

$$
\begin{array}{llllll}
\frac{6}{8} & \frac{2}{8} & \frac{5}{8} & \frac{4}{8} & \frac{1}{8} & \frac{7}{8}
\end{array} \frac{3}{8}
$$

c)

$\begin{array}{llll}\frac{3}{5} & \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\end{array}$

d) Complete the rule for comparing fractions. Circle the correct underlined word.

As the numerator gets bigger / smaller and the denominator stays the same, the fraction gets bigger/smaller/stays the same.

## Check your answers for exercise ten with your instructor.

Exercise Eleven Write the fractions on the number lines in order.
a) $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}$

b) $\frac{2}{3}, \frac{1}{3}$

c) $\frac{3}{10}, \frac{7}{10}, \frac{5}{10}, \frac{8}{10}, \frac{1}{10}$

d) $\frac{5}{6}, \frac{2}{6}, \frac{3}{6}, \frac{1}{6}$

e) $\frac{7}{12}, \frac{5}{12}, \frac{3}{12}, \frac{10}{12}, \frac{1}{12}$


Answers to Exercise Eleven
a) $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$
b) $\frac{1}{3}, \frac{2}{3}$
c) $\frac{1}{10}, \frac{3}{10}, \frac{5}{10}, \frac{7}{10}, \frac{8}{10}$
d) $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{5}{6}$
e) $\frac{1}{12}, \frac{3}{12}, \frac{5}{12}, \frac{7}{12}, \frac{10}{12}$

## Comparing Fractions with Different Denominators

You now know how to compare fractions with the same denominator, but how do you do it when the denominators of two fractions are different?

Let's look at some circles:

a) Write a fraction for each circle above.
b) Can you see that $\frac{1}{2}$ is the largest of the three fractions?

## Look at the following rectangles:

$\square$

$\qquad$
$\qquad$
$\qquad$
$\qquad$
a) Write a fraction for each rectangle
b) Can you see which is the largest? It may be harder to see as the shaded pieces are quite close in size.

Look at the following strips:

$\qquad$

a) Write in the fraction for each shaded part.
b) Write the fractions in order from the largest to the smallest:

Check with your instructor to make sure you are on the right track.

Exercise Twelve
Look back on the last three fraction drawing sets and compare the following fractions with $<$ or $>$.
a) $\frac{1}{2} \quad \frac{1}{8}$
b) $\frac{1}{4} \quad \frac{1}{3}$
c) $\frac{1}{6} \quad \frac{1}{2}$
d) $\begin{aligned} & \frac{1}{3} \quad \frac{1}{8}\end{aligned}$

## Answers to Exercise Twelve

a) $>$
b) $<$
c) $<$
d) $>$
A. Write a common fraction to describe
6 marks
i) the shaded part of each whole thing ii) the unshaded part of each whole thing
a)

b)

c)

i) shaded
i) shaded $\qquad$ i) shaded $\qquad$
ii) unshaded $\qquad$ ii) unshaded $\qquad$ i) unshaded $\qquad$
B. Draw the following fractions.

## 2 marks

a) $\frac{3}{4}$
b) $\frac{1}{5}$
C. Write the word name for the following fractions.

3 marks
a) $\frac{3}{4}$ $\qquad$
b) $\frac{1}{5}$ $\qquad$
c) $\frac{3}{7}$ $\qquad$
a) $\qquad$ The government has ordered the closing of 24 beds at the local hospital. The townspeople are angry because the hospital only has 100 beds in all. What fraction of the hospital beds are being closed?
b) The young man ordered six roses for his girlfriend. He asked for five red ones and a special yellow rose. What fraction of the roses are red?
E. Compare the following fractions, use $>$ or $<$.

3 marks
a) $\frac{4}{5}=\frac{3}{5}$
b) $\frac{12}{23}=\frac{20}{23}$
c) $\frac{1}{3} \longrightarrow \frac{2}{3}$

## Answers to Self Test

A.
a) i) $\frac{5}{8}$
ii) $\frac{3}{8}$
b) i) $\frac{2}{3}$
ii) $\frac{1}{3}$
c) i) $\frac{7}{9}$
ii) $\frac{2}{9}$
B.
a)

b)

C.
a) Three quarters or Three fourths
b) One fifth
c) Three sevenths
D.
a) $\frac{24}{100} \quad$ b) $\frac{5}{6}$
E.
a) $>$
b) $<$
c) $<$

## Topic B: Common Fractions

There are three types of fractions:

- Proper fractions are part of the whole thing.
- Improper fractions are equal to 1 or are greater than 1.
- Mixed numbers are greater than one.


## Examples:

In a proper fraction, the numerator is smaller than the denominator. Proper fractions are less than one.

$$
\begin{array}{ll}
\frac{3}{4}<1 & \frac{2}{5}<1 \\
\frac{9}{10}<1 & \frac{4}{7}<1
\end{array}
$$

In improper fractions, the numerator is the same or larger than the denominator.

$$
\begin{array}{ll}
\frac{4}{4}=1 & \frac{3}{3}=1 \\
\frac{8}{3}>1 & \frac{9}{7}>1
\end{array}
$$

In mixed numbers, a whole number and a proper fraction are used together.

$$
\begin{array}{ll}
1 \frac{1}{2}>1 & 4 \frac{3}{7}>1 \\
3 \frac{2}{5}>1 & 1 \frac{9}{10}>1
\end{array}
$$

Here are some pictures to visualize mixed numbers:


## Example A:

You want to give three small children an apple each. You need three half apples. You can write that as $\frac{3}{2}$ (three-halves).

How will you get $\frac{3}{2}$ of an apple?
You must use more than one apple. $\frac{3}{2}=1 \frac{1}{2}$ apples
This is one whole apple and $\frac{1}{2}$ of another one.


## Example B:

10 pieces of pizza are shown. Each pizza was cut into 8 pieces, so the fraction can be written as $\frac{10}{8}$. This is an improper fraction, it can also be written as a mixed number $1 \frac{2}{8}$. This is 1 whole pizza, and $\frac{2}{8}$ of another one.

## Exercise One


a) i) How many pieces of apple are shown?

ii) Each apple was cut into 4 pieces, so the denominator is $\square$
iii) Write the improper fraction that describes the photo.

iv) The photo shows
 whole apple and
 of a second apple.
v) Write the mixed number that describes the apple.


b) i) How many pieces of pizza are shown?
ii) Each pizza was cut into 8 pieces, so the denominator is

iii) Write the improper fraction that describes the photo.

iv) The photo shows
 whole pizza and of a second pizza.
v) Write the mixed number that describes the pizza.


c) i) How many pieces of pizza are shown?

ii) Each pizza was cut into 8 pieces, so the denominator is

iii) Write the improper fraction that describes the photo.

iv) The photo shows $\square$ whole pizzas and $\square$ of a third pizza.
v) Write the mixed number that describes the pizza.


Answers to Exercise One
a) i) 7
ii) 4
iii) $\frac{7}{4}$
iv) 1 and $\frac{3}{4}$
v) $1 \frac{3}{4}$
b) i) 13
ii) 8
iii) $\frac{13}{8}$
iv) 1 and $\frac{5}{8}$
v) $1 \frac{5}{8}$
c) i) 19
ii) 8
iii) $\frac{19}{8}$
iv) 2 and $\frac{3}{8}$
v) $2 \frac{3}{8}$

Exercise Two
Write the improper fraction and the mixed number that describe the shaded part in each drawing. First decide on the denominator. The denominator is what one whole thing has been divided into.


Answers to Exercise Two
b) $\frac{27}{8}=3 \frac{3}{8}$
c) $\frac{32}{6}=5 \frac{2}{6}$
d) $\frac{14}{9}=1 \frac{5}{9}$
e) $\frac{11}{4}=2 \frac{3}{4}$

Now it is your turn to draw some mixed numbers. Here is an example of how to do that. There are more shapes drawn here than you need (hint!)

## Example:

Shade in this fraction: $4 \frac{3}{4}$ (you will not need to use all the squares drawn below)


Exercise Three Shade the following mixed fractions in the given shapes.
a) $2 \frac{3}{5}$

b) $3 \frac{2}{3}$

c) $5 \frac{1}{4}$

d) $1 \frac{1}{2}$


Draw the following mixed fractions:
e) $4 \frac{1}{2}$
f) $3 \frac{4}{5}$
g) $2 \frac{2}{3}$
h) $5 \frac{3}{4}$

## Answers to Exercise Three

a)

b)

c)

d)

e)

f)

g)

h)


## Writing Improper Fractions as Mixed Numbers

In the last exercise you were able to write (rename) an improper fraction as a mixed number by looking at the drawing - you could see how many whole things were represented. You could see that if the denominator was 3 , every time you had $\frac{3}{3}$ that was one whole. If the denominator was 6 , every time you had $\frac{6}{6}$ 疗 you had one whole, and so on. An improper fraction is written (renamed) as a mixed number by dividing the numerator by the number of parts in the whole (the denominator).

To write (rename) an improper fraction as a mixed number, divide the numerator by the denominator.

Example A: $\frac{8}{5}=\square \frac{\square}{5}$

- Write as a long division problem or as a short division question.

$$
\begin{array}{lll}
\text { denominator } \longdiv { \text { numerator } } & = & 5 \sqrt{8} \\
\text { numerator } \div \text { denominator }=\square & 8 \div 5=\square
\end{array}
$$

- Divide, write the remainder as a fraction, using the same denominator.
- The 1 becomes the whole number
- Use the remainder (3) as the numerator
- Use the divisor (5) as the denominator

$$
\begin{aligned}
& 5 \longdiv { 8 } \\
& -\frac{1}{3} \quad \text { or } 8 \div 5=1 \text { remainder } 3 \text { or } 1 \mathrm{R} 3 \\
& \frac{8}{5}=1 \frac{3}{5}
\end{aligned}
$$

Example B: $\frac{16}{4}=\square \frac{\square}{4}$

$$
\begin{array}{lr}
\frac { 1 6 } { 4 } = 4 \longdiv { 1 6 } & \begin{array}{r}
4 \longdiv { 1 6 } \\
\frac{-16}{0}
\end{array} \\
\frac{16}{4}=4 &
\end{array}
$$

Exercise Four
Rename each improper fraction as an equivalent mixed number or whole number.
a) $\frac{9}{2}=4 \frac{1}{2}$
2 $\begin{array}{r}4 \\ 9 \\ \underline{8}\end{array}$
1

b) $\frac{12}{6}=2 \quad 6$| $\frac{2}{12}$ |
| :---: |
| $\frac{12}{0}$ |

c) $\quad \frac{11}{10}=$ $\qquad$
e) $\frac{17}{6}=$ $\qquad$
g) $\frac{20}{10}=$ $\qquad$
i) $\quad \frac{9}{5}=$ $\qquad$
k) $\frac{10}{5}=$ $\qquad$
m) $\frac{16}{8}=$ $\qquad$
d) $\quad \frac{12}{3}=$ $\qquad$
f) $\frac{5}{3}=$ $\qquad$
h) $\frac{12}{7}=$ $\qquad$
j) $\quad \frac{10}{3}=$ $\qquad$

1) $\frac{14}{5}=$ $\qquad$
n) $\frac{17}{4}=$ $\qquad$
o) $\frac{15}{8}=$
q) $\frac{20}{4}=$ $\qquad$ r) $\frac{11}{6}=$
p) $\quad \frac{4}{3}=$ $\qquad$
s) $\frac{13}{3}=$ $\qquad$ t) $\quad \frac{13}{6}=$ $\qquad$
u) $\quad \frac{7}{4}=$ $\qquad$
v) $\frac{5}{4}=$ $\qquad$
w) $\frac{5}{2}=$ $\qquad$
x) $\frac{10}{2}=$ $\qquad$
y) $\frac{13}{8}=$ $\qquad$ z) $\frac{12}{5}=$ $\qquad$

## Renaming Mixed Numbers as Improper Fractions

This process will be used when you multiply and divide common fractions and when you "borrow" in subtraction.

Example A: Take the whole number 2.

- Here are 2 equal shapes.

- The shapes are each divided into 3 parts (thirds).

- How many thirds are there? 6 thirds.

$$
2=\frac{6}{3}
$$

Example B: Take the whole number 1.

- Draw one shape (a circle or a box).
- Now divide the shape into half.
- How many halves are there? $\qquad$ Halves $\quad 1=\frac{2}{2}$

Example C: Take the whole number 3.

- Draw 3 shapes that are the same.
- Divide each shape into fifths (5 equal parts).
- How many fifths are there in all? $\qquad$ fifths.

$$
3=\frac{-}{5}
$$

Example D: Take the whole number 5.

- Draw 5 shapes that are the same.
- Divide each shape into fourths (4 equal parts).
- How many fourths are there in all? $\qquad$ fourths

$$
5=\frac{-}{4}
$$

## $\underset{\downarrow}{ }$ Ask your instructor to look at your examples.

## Have you found the shortcut?

The shortcut to rename a whole number as an improper fraction is to multiply the whole number by the denominator.

- $2=\frac{-}{4} \quad$ Each whole number has 4 equal parts (4 quarters) in it,

$$
\begin{aligned}
& \text { so } 2 \text { (whole numbers) } \times 4 \text { (parts) }=8 \text { parts } \\
& \text { so } 2=\frac{8}{4}
\end{aligned}
$$

- $6=\frac{-}{2}$ Each whole number has 2 equal parts ( 2 halves),

$$
\begin{aligned}
& \text { so } 6(\text { whole numbers }) \times 2(\text { parts })=12 \text { parts } \\
& \text { so } 6=\frac{12}{2}
\end{aligned}
$$

- $8=\frac{-}{3} \quad 8 \times 3=24$
so $8=\frac{24}{3}$

Until you are comfortable, draw a little sketch like you did in the example.

Exercise Five $\quad$ Rename each whole number as an improper fraction using the denominator shown.
a) $6=\frac{24}{4}$
$2=\frac{-}{3}$
$4=\frac{}{3}$
$8=\frac{}{10}$
b) $1=\frac{5}{5}$
$9=\frac{-}{8}$
$3=\frac{}{4}$
$8=\frac{-}{3}$
c) $1=\frac{-}{7}$
$2=\frac{-}{5}$
$5=\frac{}{2}$
$1=\frac{-}{8}$
d) $7=\frac{-}{3}$
$1=\frac{}{4}$
$6=\overline{10}$
$5=\frac{-}{6}$

## Answers to Exercise Five

a) $\frac{24}{4} \quad \frac{6}{3} \quad \frac{12}{3} \quad \frac{80}{10}$
b) $\frac{5}{5} \quad \frac{72}{8} \quad \frac{12}{4} \quad \frac{24}{3}$
C) $\begin{array}{llll}\frac{7}{7} & \frac{10}{5} & \frac{10}{2} & \frac{8}{8}\end{array}$
d) $\frac{21}{3} \quad \frac{4}{4} \quad \frac{60}{10} \quad \frac{30}{6}$

Now let's take this idea further. How can you rename a mixed number as an improper fraction?

## Example A:

Take the mixed number $2 \frac{1}{4}$. This is two whole things and part of a third whole thing.

- Here are three equal shapes divided into fourths.

- Shade in two whole shapes $\frac{8}{4}$ and $\frac{1}{4}$ of the third shape.
- How many fourths have you shaded in all? 9 fourths

$$
\text { So } 2 \frac{1}{4}=\frac{9}{4}
$$

## Example B:

Take the mixed number $1 \frac{2}{3}$. This is one whole thing and part of a second whole thing.

- Draw two equal shapes and divide them into thirds.
- Shade in 1 whole shape $\frac{3}{3}$ and $\frac{2}{3}$ of the second shape.
- How many thirds have you shaded in all? $\qquad$ thirds

$$
1 \frac{2}{3}=\frac{-}{3}
$$

## Example C:

Take the mixed number $4 \frac{4}{5}$. This is four whole things and part of a fifth whole thing.

- Draw five equal shapes and divide them into fifths.
- Shade in four whole shapes $\frac{20}{5}$ and $\frac{4}{5}$ of the fifth shape.
- How many fifths have you shaded in all? $\qquad$ fifths
$4 \frac{4}{5}=\frac{}{5}$

To rename a mixed number as an improper fraction, multiply the whole number by the denominator of the fraction and then add this to the numerator. Write the total as the new numerator over the denominator.

- $3 \frac{1}{4}=\frac{?}{4}$
$3=\frac{12}{4}$, so $3 \frac{1}{4}=\frac{12}{4}+\frac{1}{4}=\frac{13}{4}$
- $2 \frac{1}{2}=\frac{?}{4}$
$2=\frac{4}{2}$, so $2 \frac{1}{2}=\frac{4}{2}+\frac{1}{2}=\frac{5}{2}$


## Exercise Six

a) $2 \frac{4}{5}=\frac{14}{5}$
$8 \frac{3}{8}=\frac{-}{8}$
$7 \frac{2}{3}=\frac{-}{3}$
b) $6 \frac{5}{8}=\frac{53}{8}$
$5 \frac{1}{2}=\frac{-}{2}$
$2 \frac{5}{6}=\frac{-}{6}$
c) $8 \frac{4}{5}=$ $\qquad$ $6 \frac{1}{8}=$ $\qquad$ $7 \frac{1}{3}=$ $\qquad$
d) $5 \frac{3}{6}=$ $\qquad$
$2 \frac{7}{8}=$
$12 \frac{1}{4}=$ $\qquad$
e) $1 \frac{8}{10}=$ $\qquad$
$3 \frac{6}{8}=$ $\qquad$
$4 \frac{1}{5}=$ $\qquad$
f) $3 \frac{2}{5}=$ $\qquad$
$7 \frac{2}{3}=$ $\qquad$
$4 \frac{3}{4}=$ $\qquad$

## Answers to Exercise Six

a) $\frac{14}{5} \quad \frac{67}{8} \quad \frac{23}{3}$
b) $\frac{53}{8} \quad \frac{11}{2} \quad \frac{17}{6}$
c) $\frac{44}{5} \quad \frac{49}{8} \quad \frac{22}{3}$
d) $\frac{33}{6} \quad \frac{23}{8} \quad \frac{49}{4}$
e) $\frac{18}{10} \quad \frac{30}{8} \quad \frac{21}{5}$
f) $\frac{17}{5} \quad \frac{23}{3} \quad \frac{19}{4}$

## To Say or Write a Mixed Number

If you want to say these crazy looking fractions out loud, you do as follows:

Write or say:

1) the whole number
2) and
3) the fraction

Example: $3 \frac{2}{5}=$ three and two fifths
$5 \frac{1}{6}=$ five and one sixth

Exercise Seven
Write the following fractions as words.
a) $2 \frac{1}{8}$
b) $4 \frac{3}{4}$
c) $1 \frac{1}{2}$
d) $3 \frac{3}{4}$
e) $7 \frac{3}{8}$
f) $8 \frac{4}{5}$
g) $\quad 9 \frac{3}{8}$
h) $2 \frac{5}{6}$
i) $\quad 12 \frac{1}{4}$

## Answers to Exercise Seven

a) two and one eighth
b) four and three fourths (or four and three quarters)
c) one and one half
d) three and three fourths (or three and three quarters)
e) seven and three eighths
f) eight and four fifths
g) nine and three eighths
h) two and five sixths
i) twelve and one fourth (or twelve and one quarter)

## Topic B Self-Test

Mark $/ 16$ Aim 13/16
A. Write the improper fraction and the mixed number that 4 marks
describe the shaded part of the drawings in each question.
a)

b)

B. Rename each improper fraction as an equivalent

6 marks mixed number or whole number.
a) $\frac{9}{4}=$ $\qquad$
b) $\frac{12}{3}=$ $\qquad$
c) $\quad \frac{22}{5}=$ $\qquad$
d) $\frac{3}{2}=$ $\qquad$
e) $\frac{5}{3}=$ $\qquad$
f) $\quad \frac{7}{2}=$ $\qquad$
C. Rename as improper fractions.

6 marks
a) $3 \frac{1}{2}=$ $\qquad$ b) $4 \frac{3}{8}=$ $\qquad$
c) $5 \frac{3}{4}=$ $\qquad$ d) $\quad 1=\frac{?}{2}=$ $\qquad$
e) $1 \frac{7}{8}=$ $\qquad$
f) $2 \frac{5}{6}=$ $\qquad$

## Answers to Topic B Self Test

A.
a) $\frac{10}{3}=3 \frac{1}{3}$
b) $\frac{7}{4}=1 \frac{3}{4}$
B.
a) $2 \frac{1}{4}$
b) 4
c) $4 \frac{2}{5}$
d) $1 \frac{1}{2}$
e) $1 \frac{2}{3}$
f) $3 \frac{1}{2}$
C.
a) $\frac{7}{2}$
b) $\frac{35}{8}$
c) $\frac{23}{4}$
d) $1=\frac{2}{2}$
e) $\frac{15}{8}$
f) $\frac{17}{6}$

## Topic C: When to Use a Fraction or a Decimal

Sometimes you need to decide to write or say a number as a fraction or a decimal. We usually choose the most common method, which is usually the easiest way to say something. It may seem to you that nothing is easy at this moment, so here are some tips.

When we talk about money, we almost always talk about a part of a dollar.

Example: Two dollars and fifteen cents is two whole dollars and fifteen parts of one more dollar.
$\$ 2.15$ or $\$ 2 \frac{15}{100}$ - which of these ways of writing money is more common, or easy to you?

We usually write or talk about money in decimals:
Example 1: $\$ 2.50$ (two dollars and fifty cents) instead of $\$ 2 \frac{1}{2}$ (two and a half dollars)
Example 2: $\$ 0.50$ (fifty cents) instead of $\$ \frac{1}{2}$ (half a dollar)
Example 3: $\$ 67.30$ (sixty seven dollars and thirty cents) instead of $\$ 67 \frac{30}{100}$ (sixty seven dollars and thirty hundredths cents)

But: There is one place where we talk about money as a fraction: The quarter! A quarter equals $\$ 0.25$, but we often say "it costs a quarter" as much as we say "it costs 25 cents".

Really, if we were speaking correctly, it would be a quarter of a dollar, but it gets shortened. Also, we still write $\$ 0.25$ or $25 \phi$, not $\$ \frac{25}{100}$ or $\$ \frac{1}{4}$.

In most other ways of talking and writing, fractions and decimals are expressed in what seems easiest. This means that you get the say the number in the way you like best.

Example: Saying six point four grams may be faster and easier than saying six and two fifths grams.

But, saying $\frac{3}{4}$ of a tank of gas makes more sense than saying 0.75 of a tank of gas.

Circle the way that you think a number should be said out loud. (read the numbers out loud with a friend to help hear them) Remember, that sometimes your answers will be different from the ones in the answer key because you have a different opinion, and that it O.K.

| a) | $\$ 12.25$ | or | $\$ 12 \frac{1}{4}$ |
| :--- | :--- | :--- | :--- |
| b) | 51.4 cm | or | $51 \frac{2}{5} \mathrm{~cm}$ |
| c) | One million two hundred thousand <br> dollars | or | One point two million dollars |
| d) | 563.56 km | or | $563 \frac{56}{100} \mathrm{~km}$ |
| e) | $5 \frac{7}{10} \mathrm{~L}$ | or | 5.7 L |
| f) | I ran a tenth of a kilometer | or | I ran zero point one kilometers |
| g) | It weighs five and a half grams | or | It weighs five point five grams |
| h) | $\$ 39 \frac{99}{100}$ | or | $\$ 39.99$ |

## Answers to Exercise One

a) $\$ 12.25$
b) 51.4 cm
c) One point two million dollars
d) 563.56 km
e) 5.7 L
f) I ran a tenth of a kilometer
g) both
h) $\$ 39.99$

## Unit One Review

1. Write fractions from the pictures
a)

b)

c)

d)

e)

f)

2. Draw your own fractions:
a) $\frac{2}{5}$
b) $\frac{3}{4}$
c) $\frac{4}{9}$
d) $\frac{1}{6}$
e) $\frac{1}{3}$
f) $\frac{7}{10}$
3. Write the following fractions in words:
a) $\frac{1}{2}$ one half
b) $\frac{1}{4} \xrightarrow{\text { one quarter }}$
c) $\frac{3}{4}$
d) $\frac{2}{5}$
e) $\frac{4}{9}$
f) $\frac{2}{3}$
g) $\frac{7}{10}$
h) $\frac{21}{25}$
i) $\frac{1}{3}$
j) $\frac{5}{6}$
$\qquad$
4. Answer the questions using a common fraction:
a) Suzie jogged 20 minutes out of 1 hour. What fraction of the hour did she jog? (remember 1 hour $=60$ minutes)
b) Oliver planted 30 garlic cloves in September. 25 shoots have come up in the spring. What fractions of garlic bulbs did not grow a shoot?
c) The class usually had 8 students, but 6 did not come on Monday. What fraction of students did not come?
d) Stephen made 60 Easter cookies. His brother ate 3, his mom ate 3 more. What fraction of cookies were eaten by Stephen's family?
e) Thrifty's grocery store sold 300 dozen eggs in one week. They had 450 dozen in stock. What fraction of the stock was sold?
f) The test was out of 32 . Sasha got 30 marks. What was her score?
5. Compare the following fractions:
a) $\frac{3}{4}-\frac{1}{4}$
b) $\frac{9}{10}-\frac{3}{10}$
c) $\frac{1}{5}=\frac{3}{5}$
d) $\frac{3}{8}-\frac{7}{8}$
e) seven tenths $\qquad$ three tenths
f) one quarter $\qquad$ three quarters
g) four fifths $\qquad$ three fifths
h) one twelfth $\qquad$ eleven twelfths
6. Identify each fraction by writing: proper fraction, improper fraction, or mixed number next to each fraction.
a) $\frac{1}{2}$
b) $\frac{3}{2}$
c) $\frac{100}{47}$
d) $1 \frac{3}{7}$ $\qquad$
e) $\frac{5}{6}$
f) $3 \frac{1}{4}$ $\qquad$
g) $\frac{51}{2}$ $\qquad$
h) $\frac{1}{3}$ $\qquad$
i) $42 \frac{1}{4}$ $\qquad$
j) $\frac{4}{3}$ $\qquad$
7. Fill in the missing parts of the chart:

|  | Improper Fraction | Mixed Number | Drawing |
| :---: | :---: | :---: | :---: |
| a) |  |  | $0000$ |
| b) | $\frac{29}{6}$ |  |  |
| c) |  | $2 \frac{3}{4}$ |  |
| d) |  |  | $\square \square \square \square \square \square \square \square \square \square \square \square \square \square \square \square \square \square \square$ $\square \square \square \square \square \square \square \square \square \square \square \square \square \square \square \square$ |
| e) | $\frac{9}{2}$ |  |  |
| f) |  | $3 \frac{1}{5}$ |  |
| g) |  |  |  |
| h) | $\frac{25}{8}$ |  |  |
| i) |  | $3 \frac{1}{3}$ |  |

8. Rename each improper fraction into a mixed number or a whole number.
a) $\frac{9}{5}=$ $\qquad$
b) $\quad \frac{7}{2}=$
c) $\quad \frac{11}{2}=$ $\qquad$
d) $\frac{14}{5}=$ $\qquad$
e) $\frac{4}{3}=$ $\qquad$
f) $\frac{11}{6}=$ $\qquad$
g) $\frac{14}{3}=$ $\qquad$
h) $\frac{10}{2}=$ $\qquad$
i) $\frac{13}{5}=$ $\qquad$
j) $\frac{11}{8}=$ $\qquad$
k) $\quad \frac{6}{6}=$ $\qquad$
1) $\frac{5}{4}=$
$\qquad$
m) $\frac{7}{3}=$ $\qquad$
n) $\frac{8}{4}=$ $\qquad$
9. Rename each whole number as an improper fraction. Use the denominator given to you.
a) $6=\frac{-}{3}$
b) $\quad 5=\frac{-}{2}$
c) $3=\frac{-}{7}$
d) $4=\frac{-}{5}$
e) $\quad 1=\frac{-}{1}$
f) $9=\frac{-}{2}$
g) $7=\frac{-}{3}$
h) $8=\frac{7}{7}$
i) $\quad 1=\frac{-}{3}$
j) $2=\frac{}{10}$
10. Rename each mixed number as an improper fraction.
a) $6 \frac{7}{8}=$ $\qquad$ b) $2 \frac{1}{2}=$ $\qquad$ c) $\quad 15 \frac{4}{5}=$ $\qquad$
d) $2 \frac{1}{3}=$ $\qquad$
e) $\quad 9 \frac{9}{10}=$ $\qquad$
f) $4 \frac{1}{6}=$ $\qquad$
g) $20 \frac{3}{7}=$ $\qquad$
h) $\quad 18 \frac{1}{2}=$ $\qquad$
i) $\quad 5 \frac{9}{11}=$ $\qquad$
j) $3 \frac{1}{4}=$ $\qquad$

## Answers to Review

1. 

a) $\frac{1}{4}$
b) $\frac{2}{3}$
c) $\frac{3}{4}$
d) $\frac{1}{2}$
e) $\frac{2}{4} \quad$ f) $\frac{4}{6}$
2.

a)
c)

d)

e)

f)

3.
a) One half
b) One fourth or one quarter
c) Three fourths or three quarters
d) Two fifths
e) Four ninths
f) Two thirds
g) Seven tenths
h) Twenty one twenty fifths
i) One third
j) Five sixths
4.
a) $\frac{20}{60}$
b) $\frac{5}{30}$
c) $\frac{6}{8}$
d) $\frac{6}{60}$
e) $\frac{300}{450}$
f) $\frac{30}{32}$
5.
a) $>$
b) $>$
c) $<$
d) $<$
e) $>$
f) $<$
g) $>$
h) $<$
6.
a) proper fraction
b) improper fraction
c) improper fraction
d) mixed number
e) proper fraction
f) mixed number
g) improper fraction
h) proper fraction
i) mixed number
j) improper fraction

## 7.

|  | Improper Fraction | Mixed Number | Drawing |
| :---: | :---: | :---: | :---: |
| a) | $\frac{11}{3}$ | $3 \frac{2}{3}$ | $\theta \otimes \theta$ |
| b) | $\frac{29}{6}$ | $4 \frac{5}{6}$ | $\otimes \otimes \otimes \otimes$ |
| c) | $\frac{11}{4}$ | $2 \frac{3}{4}$ | $\because \exists \exists$ |
| d) | $\frac{31}{10}$ | $3 \frac{1}{10}$ | $\square \square \square \square \square \square \square \square \square \square \square \square \square \square ~$ $\square \square \square \square \square \square \square \square \square \square \square$ |
| e) | $\frac{9}{2}$ | $4 \frac{1}{2}$ | $\stackrel{\rightharpoonup}{\rho} \stackrel{\rightharpoonup}{ }$ |
| f) | $\frac{16}{5}$ | $3 \frac{1}{5}$ |  |
| g) | $\frac{11}{9}$ | $1 \frac{2}{9}$ |  |
| h) | $\frac{25}{8}$ | $3 \frac{1}{8}$ | $\sqrt[4]{4} \sqrt{s}$ |
| i) | $\frac{10}{3}$ | $3 \frac{1}{3}$ |  |

8. 

a) $1 \frac{4}{5}$
b) $3 \frac{1}{2}$
c) $5 \frac{1}{2}$
d) $2 \frac{4}{5}$
e) $1 \frac{1}{3}$
f) $1 \frac{5}{6}$
g) $4 \frac{2}{3}$
h) 5
i) $2 \frac{3}{5}$
j) $1 \frac{3}{8}$
k) 1

1) $1 \frac{1}{4}$
m) $2 \frac{1}{3}$
n) 2
9. 

a) $\frac{18}{3}$
b) $\frac{10}{2}$
c) $\frac{21}{7}$
d) $\frac{20}{5}$
e) $\frac{1}{1}$
f) $\frac{18}{2}$
g) $\frac{21}{3}$
h) $\frac{56}{7}$
i) $\frac{3}{3}$
j) $\frac{20}{10}$
10.
a) $\frac{55}{8}$
b) $\frac{5}{2}$
c) $\frac{79}{5}$
d) $\frac{7}{3}$
e) $\frac{99}{10}$
f) $\frac{25}{6}$
g) $\frac{143}{7}$
h) $\frac{37}{2}$
i) $\frac{64}{11}$
j) $\frac{13}{4}$

## It is now test time!

Please get the practice test from your instructor.

Once you are ready, you can get the unit 1 test from your instructor.

## Good luck!

