## Unit 5

## Common Fractions \& Decimals

## Topic A: Common Fractions \& Decimals

The amount represented by a fraction may be expressed as a common fraction, a decimal, or as a percent.

We choose common fractions, decimals, or percents for convenience and to fit the standard way of doing things.

## Common fractions are used:

- For everyday conversation about parts of the whole thing ( $\frac{1}{2}$ cup of coffee, $\frac{1}{4}$ of an hour, $\frac{3}{4}$ tank of gas)
- with amounts in the Imperial System of measurement which is standard in the United States and still used by some people in Canada
( $3 \frac{1}{4}$ feet, $\frac{5}{8}$ inches, $12 \frac{3}{4}$ miles, $6 \frac{1}{4}$ pounds, $1 \frac{1}{2}$ teaspoons)
- For stock market reports and stock values
- For the score on the top of a test (which is usually changed to a percent)


## Decimals are used

- with money (\$12.23)
- with the metric system of measurement (1.5 metres, 7.25 litres, 29.75 kilometres, 0.5 centimetres, 9.2 grams, 75.5 kilograms, etc.)
- whenever there is a lot of arithmetic calculation to be done
- for calculators and computers


## Percents are used

- for reporting statistics
- for bank rates and interest charges such as mortgage rates
- for reading a grade on a test


## Writing Decimals as Common Fractions

Remember this skill?
a) $0.48=\frac{48}{100}$
b) $3.542=3 \frac{542}{1000}$

Common fractions should always be in lowest terms.
a) $0.48=\frac{48}{100}\left(\frac{\div 4}{\div 4}\right)=\frac{12}{25}$
b) $3.542=3 \frac{542}{1000}\left(\frac{\div 2}{\div 2}\right)=3 \frac{271}{500}$

This list of factors may help you to simplify the fractions.

The factors of $\mathbf{1 0}$ are 1, 2, 5, 10

The factors of $\mathbf{1 0 0}$ are $1,2,4,5,10,20,25,50,100$

The factors of $\mathbf{1 0 0 0}$ are $1,2,5,8,10,20,25,50,100,125,200,250,500,1000$

Remember: the whole number in a mixed decimal stays a whole number in a mixed fraction.

Exercise One
Write these decimals as common fractions expressed in lowest terms.
a) $\quad 16.04=16 \frac{4}{100} \frac{\div 4}{\div 4}=16 \frac{1}{25}$
b) $\quad 0.085=\frac{85}{1000} \frac{\div 5}{\div 5}=\frac{17}{200}$
c) $\quad 3.48=$ $\qquad$ d) $\quad 12.075=$ $\qquad$
e) $\quad 6.25=$ $\qquad$
f) $\quad 25.025=$
g) $\quad 9.500=$ $\qquad$ h) $5.6=$
i) $\quad 0.07=$ $\qquad$
j) $\quad 0.14=$ $\qquad$
k) $\quad 12.125=$ $\qquad$ 1) $1.75=$ $\qquad$

## Answers to Exercise One

c) $3 \frac{12}{25}$
d) $12 \frac{3}{40}$
e) $6 \frac{1}{4}$
f) $25 \frac{1}{40}$
g) $9 \frac{1}{2}$
h) $5 \frac{3}{5}$
i) $\frac{7}{100}$
j) $\frac{7}{50}$
k) $12 \frac{1}{8}$

1) $1 \frac{3}{4}$

## Some Tricky Conversions

Do you remember that there are some fractions that do not convert into decimals perfectly? The reason they do not is because they have a repeating decimal.

## Some are:

$\frac{1}{6}=0 . \overline{1} \quad ; \quad \frac{1}{3}=0 . \overline{3} \quad ; \quad \frac{2}{3}=0 . \overline{6} \quad ; \quad \frac{5}{6}=0.8 \overline{3} \quad ; \frac{1}{9}=0 . \overline{1}$

Because of this, it is not possible to convert a repeating decimal into a fraction perfectly either. It is not possible to convert $0.1 \overline{6}$ (which is really $0.166666666666 \overline{6}$ ) into a fraction.

The only way to deal with this problem is: to remember that there are some tricky fractions that do not convert into perfect decimals.

## Writing Common Fractions as Decimals

As you know, common fractions with denominators of $10,100,1000$, or 10000 are easily written as decimals.

$$
\begin{aligned}
& \frac{3}{10}=0.3 \\
& \frac{21}{100}=0.21 \\
& \frac{69}{1000}=0.069
\end{aligned}
$$

But if the denominator is not a 10,100 , etc., you may be able to change a common fraction to an equivalent fraction with a denominator of $10,100,1000$, or 10000 which can then be written easily as a decimal. For example,

$$
\begin{aligned}
& \frac{3}{5}\left(\frac{\times 2}{\times 2}\right)=\frac{6}{10}=0.6 \\
& \frac{1}{2}\left(\frac{\times 5}{\times 5}\right)=\frac{5}{10}=0.5 \\
& \frac{4}{25}\left(\frac{\times 4}{\times 4}\right)=\frac{16}{100}=0.16
\end{aligned}
$$

## Exercise Two

a) $\frac{1}{2}=\frac{5}{10}=0.5$
b) $\frac{2}{5}=$
c) $\quad \frac{7}{10}=$ $\qquad$
d) $\frac{4}{5}=$ $\qquad$
e) $\frac{75}{100}=$ $\qquad$
f) $\frac{3}{50}=$ $\qquad$
h) $\frac{8}{25}=$ $\qquad$
Answers to Exercise Two
b) 0.4
c) 0.7
d) 0.8
e) 0.75
f) 0.06
g) 0.021
h) 0.32

Here is a review of how to change a fraction to a decimal when it is not easy to make the denominator:

The line in a common fraction can be thought of as a divided by sign $\div$ To change a common fraction to a decimal, do this:
numerator $\div$ denominator $=$ the decimal equivalent

Example A: $\frac{3}{4} \quad$ Think $3 \div 4$

$$
\begin{array}{r}
0.75 \\
4 \begin{array}{r}
3.00 \\
28 \downarrow \\
20 \\
20
\end{array}
\end{array} \quad \frac{3}{4}=0.75
$$

Example B: $\frac{3}{8} \quad$ Think $3 \div 8$

$$
\begin{array}{r}
0.375 \\
8 \longdiv { 3 . 0 0 0 } \\
24 \downarrow \\
60 \\
56 \downarrow \\
\hline 40 \\
40
\end{array} \quad \frac{3}{8}=0.375
$$

Example C: $\frac{1}{3} \quad$ Think $1 \div 3$

$$
\begin{array}{r}
3 \begin{array}{r}
0.333 \\
1.000 \\
9 \downarrow \\
10 \\
0 \\
0 \\
10
\end{array} \quad \frac{1}{3}=0.333 \\
\\
\end{array}
$$

Exercise Three
a) $\begin{aligned} & \frac{1}{2} \\ & \frac{1}{2}=0.5\end{aligned}$
c) $\frac{2}{5}$
d) $\frac{6}{12}$
e) $\frac{1}{8}$
f) $\frac{3}{8}$
g) $\frac{5}{12}$
h) $\frac{1}{6}$
i) $\frac{2}{3}$
j) $\frac{19}{20}$

Answers to Exercise Three
b) 0.25
c) 0.4
d) 0.5
e) 0.125
f) 0.375
g) $0.4 \overline{6}$
h) $0.1 \overline{6}$
i) $0 . \overline{6}$
j) 0.95

The whole number in a mixed fraction stays a whole number in a mixed decimal. Rewrite the whole number to the left of the decimal. Then change the common fraction to a decimal.

$$
\begin{array}{ll}
4 \frac{3}{4}=4.75 & \text { Think } \frac{3}{4}=3 \div 4=0.75 \\
16 \frac{1}{2}=16.5 & \text { Think } \frac{1}{2}=1 \div 2=0.5
\end{array}
$$

Complete the chart of equivalent common fractions and decimals. Use this chart as a reference for yourself in later work. Look for patterns that develop and note them in the margin.

| Common Fraction | Decimal |
| :--- | :--- |
| $\frac{1}{8}$ |  |
| $\frac{2}{8}=\frac{1}{4}$ |  |
| $\frac{3}{8}$ |  |
| $\frac{4}{8}=\frac{2}{4}=\frac{1}{2}$ |  |
| $\frac{5}{8}$ |  |
| $\frac{6}{8}=\frac{3}{4}$ |  |
| $\frac{7}{8}$ |  |
| $\frac{4}{12}=\frac{2}{6}=\frac{1}{3}$ |  |
| $\frac{1}{12}=\frac{4}{4}=\frac{2}{2}=1$ |  |
| $\frac{1}{6}$ |  |


| Common Fraction | Decimal |
| :--- | :--- |
| $\frac{6}{12}=\frac{3}{6}=\frac{1}{2}$ |  |
| $\frac{8}{12}=\frac{4}{6}=\frac{2}{3}$ |  |
| $\frac{10}{12}=\frac{5}{6}$ |  |
| $\frac{12}{12}=\frac{6}{6}=\frac{3}{3}$ |  |
| $\frac{1}{20}$ |  |
| $\frac{2}{20}=\frac{1}{10}$ |  |
| $\frac{2}{10}=\frac{1}{5}$ |  |
| $\frac{4}{10}=\frac{2}{5}$ |  |
| $\frac{6}{10}=\frac{3}{5}$ |  |
| $\frac{8}{10}=\frac{4}{5}$ |  |
| 10 |  |

## Answers to Exercise Four

| Common <br> Fraction | Decimal |
| :--- | :--- |
| $\frac{1}{8}$ | 0.125 |
| $\frac{2}{8}=\frac{1}{4}$ | 0.25 |
| $\frac{3}{8}$ | 0.375 |
| $\frac{4}{8}=\frac{2}{4}=\frac{1}{2}$ | 0.5 |
| $\frac{5}{8}$ | 0.625 |
| $\frac{6}{8}=\frac{3}{4}$ | 0.75 |
| $\frac{7}{8}$ | 0.875 |
| $\frac{8}{8}=\frac{4}{4}=\frac{2}{2}=1$ | 1.0 |
| $\frac{1}{12}$ | $0.08 \overline{3}$ |
| $\frac{2}{12}=\frac{1}{6}$ | $0.1 \overline{6}$ |
| $\frac{4}{12}=\frac{2}{6}=\frac{1}{3}$ | $0 . \overline{3}$ |
| $\frac{6}{12}=\frac{3}{6}=\frac{1}{2}$ | 0.5 |


| Common <br> Fraction | Decimal |
| :--- | :--- |
| $\frac{8}{12}=\frac{4}{6}=\frac{2}{3}$ | $0 . \overline{6}$ |
| $\frac{10}{12}=\frac{5}{6}$ | $0.8 \overline{3}$ |
| $\frac{12}{12}=\frac{6}{6}=\frac{3}{3}$ | 1.0 |
| $\frac{1}{20}$ | 0.05 |
| $\frac{2}{20}=\frac{1}{10}$ | 0.1 |
| $\frac{2}{10}=\frac{1}{5}$ | 0.2 |
| $\frac{4}{10}=\frac{2}{5}$ | 0.4 |
| $\frac{6}{10}=\frac{3}{5}$ | 0.6 |
| $\frac{8}{10}=\frac{4}{5}$ | 0.8 |
| $\frac{10}{10}=\frac{5}{5}=1$ | 1.0 |

You may work with problems and real-life situations that use one decimal and one common fraction. Rewrite the fractions so both are decimals or both are common fractions. Choose the fraction form that will give the answer the way it should be written.

Example A: Ted worked $3 \frac{3}{4}$ hours at $\$ 8.25$ per hour. How much did he earn?
(Round to the nearest cent.)
The answer will be money which should be written using decimals, so work in the decimal form.

Rewrite $3 \frac{3}{4}$ hours as 3.75 hours.
Ted earned $3.75 \times \$ 8.25=\$ 30.94$

Example B: Jane cycled 49.4 km in $2 \frac{1}{2}$ hours. What was her average speed?
The answer will be in $\mathrm{km} / \mathrm{hr}$. Metric measurements are written with decimals, so work in decimals.

Rewrite $2 \frac{1}{2}$ hours as 2.5 hours and solve the problem.
$49.4 \div 2.5=19.76 \mathrm{~km} / \mathrm{hr}$.

## A. Complete the chart.

|  | Common Fraction | Decimal |
| :--- | :---: | :--- |
| a) | $\frac{1}{4}$ |  |
| b) |  | 0.125 |
| c) |  | $0 . \overline{3}$ |
| d) | $\frac{3}{4}$ |  |
| e) |  | 0.875 |
| f) | $\frac{3}{5}$ |  |
| g) | $\frac{6}{6}$ |  |

## B. Answer the following word problems.

a) Joseph worked $5 \frac{3}{4}$ hours a day, 5 days a week. He gets paid $\$ 9.35$ per hour. How much does he get paid a week?
b) Giang ran a 42.195 km marathon in $4 \frac{1}{4}$ hours. What was her average speed rounded to two decimal places?

## Answers to Self Test

A.

|  | Common Fraction | Decimal |
| :--- | :---: | :--- |
| a) | $\frac{1}{4}$ | 0.25 |
| b) | $\frac{1}{8}$ | 0.125 |
| c) | $\frac{1}{3}$ | $0 . \overline{3}$ |
| d) | $\frac{3}{4}$ | 0.75 |
| e) | $\frac{7}{8}$ | 0.875 |
| f) | $\frac{3}{5}$ | 0.6 |
| g) | $\frac{6}{6}$ | 1 |

B. a) $\$ 268.81$
b) $9.93 \mathrm{~km} / \mathrm{hr}$

## Topic B: Comparing Fractions and Decimals

In Unit 2 you compared simple fractions to simple fractions such as $\frac{1}{2}-\frac{1}{3}$.
Now, you are going to be asked to compare fractions with larger and less common denominators such as $\frac{7}{8}$ and $\frac{17}{28}$.

Find the equivalent fractions of both the fractions and then compare.

$$
\begin{aligned}
& \frac{1}{2}\left(\frac{\times 3}{\times 3}\right)=\frac{3}{6} \longrightarrow \frac{1}{3}\left(\frac{\times 2}{x 2}\right)=\frac{2}{6} \longrightarrow \frac{3}{6} \geq \frac{2}{6} \\
& \frac{7}{8}\left(\frac{\times 7}{\times 7}\right)=\frac{49}{56} \longrightarrow \frac{17}{28}\left(\frac{\times 2}{x 2}\right)=\frac{34}{56} \longrightarrow \frac{49}{56} \geq \frac{34}{56}
\end{aligned}
$$

Exercise One
Change the fractions to have the same denominators. Decide which is larger. Use $>,<,=$ to mark your answer. There is space below each problem to work out your equivalent fractions.
a) $\frac{1}{4} \longrightarrow \frac{3}{5}$
b) $\frac{5}{8} \longrightarrow \frac{3}{4}$
c) $\frac{1}{5} \longrightarrow \frac{2}{3}$
d) $\frac{4}{5} \longrightarrow \frac{3}{4}$
e) $\frac{2}{3} \longrightarrow \frac{3}{4}$
f) $\frac{5}{8} \longrightarrow \frac{3}{5}$
g) $\frac{49}{56}=\frac{3}{4}$
h) $\frac{75}{90} \longrightarrow \frac{10}{11}$
i) $\frac{3}{5} \longrightarrow \frac{1}{3}$
j) $\frac{33}{70}-\frac{1}{2}$
k) $\frac{14}{15}-\frac{9}{10}$

1) $\frac{1}{3} \longrightarrow \frac{1}{9}$

## Answers to Exercise One

a) $<$
b) $<$
c) $<$
d) $>$
e) $<\quad$ f) $<$
g) $>$
h) $<$
i) $>$
j) $<$
k) $>$

1) $>$

## Comparing Fractions to Decimals and Decimals to Fractions

Now that you know how to convert between fractions and decimals, you are also going to be able to compare them.

## Example A:

Which is larger: $\frac{3}{8}$ or 0.125 ?

1) Decide to convert either the fraction into a decimal, or the decimal into a fraction.

Let's pick converting the fraction to a decimal.
2) Do the conversion: $\frac{3}{8}=$

3) Compare 0.375 $\qquad$ 0.125
4) Write a math sentence to show your answer: $\frac{3}{8} \geq 0.125$

## Example B:

Which is larger $\frac{7}{8}$ or 0.6 ?

1) Pick which you will convert, the decimal into the fraction, or the fraction into the decimal. Let's pick converting the decimal this time.
2) Convert $0.6=\frac{6}{10}\left(\frac{\div 2}{\div 2}\right)=\frac{3}{5}$
3) Compare $\frac{7}{8} \longrightarrow \frac{3}{5} \longrightarrow \frac{7}{8}\left(\frac{x 5}{x 5}\right)=\frac{35}{40} \longrightarrow \frac{3}{5}\left(\frac{x 8}{\times 8}\right)=\frac{24}{40}$
4) Write a math sentence to show your answer: $\frac{7}{8} \_\geq 0.6$

You may find that converting the fractions to the decimals could speed things up.
a) $0.25=\frac{1}{5}$
b) $\frac{1}{8} \longrightarrow 0.8$
c) $\frac{1}{6} \longrightarrow 0.125$
d) $0.875=\frac{5}{6}$
e) $0.625=\frac{3}{5}$
f) $\frac{4}{5} \longrightarrow 0.75$
g) $0.2=\frac{3}{10}$
h) $0.375=\frac{2}{3}$
i) $\frac{2}{5} \longrightarrow 0.5$
j) $0.375=\frac{1}{5}$

## Answers to Exercise Two

a) $>$
b) $<$
c) $>$
d) $>$
e) $>$
f) $>$
g) $<$
h) $<$
i) $<$
j) $>$

Exercise Three
a) $0.345-\frac{6}{7}$
b) $\frac{32}{71}$ $\qquad$ 0.42
c) $\frac{1}{1051}-0.0032$
d) $0.337-\frac{5}{26}$
e) $0.52=\frac{10}{21}$
f) $\frac{75}{90}$
g) $0.6894=\frac{103}{278}$
h) $0.9993=\frac{44}{47}$
i) $\frac{1}{72} \quad 0.034$

ј) $0.5642 \longrightarrow \frac{5}{9}$

Answers to Exercise Three
a) $<$
b) $>$
c) $<$
d) $>$
e) $>$ f) $>$
g) $>$
h) $>$
i) $<$ j) $>$

## Topic B Self-Test

Mark $\quad / 18 \quad$ Aim 15/18
A. Write as common fractions in lowest terms.
a) 3.6
b) 8.125
c) 0.75
d) 0.45
B. Write as decimals. Round your answers to 3 decimal places.

4 marks
a) $\frac{1}{3}$
b) $\frac{6}{100}$
c) $\frac{3}{8}$
d) $\frac{5}{7}$
C. Compare the following fractions and decimals, use $<$, or $>$.
a) $0.862=\frac{6}{7}$
b) $\frac{3}{14}$ 0.35
c) $\frac{1}{10}-0.007$
d) $0.3-\frac{20}{26}$
e) $0.5-\frac{10}{21}$
f) $\frac{89}{90}$ 0.473
D. Problems 4 marks
a) May babysat for $4 \frac{1}{4}$ hours. She is paid $\$ 4.75$ an hour. How much did she earn?
b) Diane bought 1.5 litres of milk. Her thirsty kids drank half of it as soon as she got home. How much milk is left?

## Answers to Topic B Self-Test

Part A
a) $3 \frac{3}{5}$
b) $8 \frac{1}{8}$
c) $\frac{3}{4}$
d) $\frac{9}{20}$

Part B
a) $0 . \overline{3}$
b) 0.06
c) 0.375
d) 0.714

Part C
a) $>$
b) $<$
c) $>$
d) $<$
e) $>$
f) $>$

## Part D

a) $\$ 20.19$
b) 0.75 L

## Unit 5 Review

1) Write as common fractions in lowest terms.
a) 3.5
b) 8.625
c) 0.25
d) 0.375
e) 4.125
f) 10.4
g) 2.6
h) 3.05
2) Write as decimals. Round your answers to 3 decimal places.
a) $\frac{2}{3}$
b) $\frac{12}{100}$
c) $\frac{5}{8}$
d) $\frac{6}{7}$
e) $\frac{3}{8}$
f) $\frac{1}{6}$
g) $\frac{17}{25}$
h) $\frac{5}{6}$
i) $\frac{1}{5}$
j) $\frac{1}{3}$
3) Compare the following fractions to fractions, use $<$, or $>$.
a) $\frac{1}{4} \longrightarrow \frac{6}{7}$
b) $\frac{3}{14}-\frac{7}{49}$
c) $\frac{1}{10} \longrightarrow \frac{6}{30}$
d) $\frac{9}{13}-\frac{20}{26}$
e) $\frac{1}{3} \longrightarrow \frac{10}{21}$
f) $\frac{8}{9} \longrightarrow \frac{6}{7}$
4) Compare the following fractions to decimals. Use $>,<$, or $=$
a) $0.8=\frac{6}{7}$
b) $\frac{3}{7}$
0.52
c) $\frac{1}{5}$
0.20
d) $0.125=\frac{5}{8}$
e) $0.63-\frac{7}{25}$
f) $\frac{65}{90} \longrightarrow 0.4$
g) $0.45-\frac{19}{27}$
h) $0.99=\frac{1}{4}$
i) $\frac{1}{7} \longrightarrow 0.39$
j) $0.375=\frac{3}{8}$

## Answers to Unit 5 Review

1) 

a) $3 \frac{1}{2}$
b) $8 \frac{5}{8}$
c) $\frac{1}{4}$
d) $\frac{3}{8}$
e) $4 \frac{1}{8}$
f) $10 \frac{2}{5}$
g) $2 \frac{3}{5}$
h) $3 \frac{1}{20}$
2)
a) 0.667
b) 0.12
c) 0.625
d) 0.857
e) 0.375
f) 0.167
g) 0.68
h) $0.8 \overline{3}$
i) 0.2
j) $0 . \overline{3}$
3)
a) $<$
b) $>$
c) $<$
d) $<$
e) $<$
f) $>$
4)
a) $<$
b) $<$
c) $=$
d) $<$
e) $>$
f) $>$
g) $<$
h) $>$
i) $<$
j) $=$

## It is now Test time.

Please see your instructor to get the practice test.

Once you are prepared, you can write the unit 5 test.
Your instructor will get that for you too!

Once you have completed the unit 6 test, it will be time to write the final test.

## Good luck!

