

Unit 1

Ratio, Rate & Proportion

Introduction

Unit 1 will introduce ratios, rates and proportions.

- Using **ratios** we can compare quantities and see the relationship between the numbers in a simple way.
- **Rates** are used when a ratio is comparing two *different* kinds of measure.
- A **Proportion** is a statement with two equal ratios. They are be very useful in solving per cent problems

Unit 2 prepares you to work with per cents. You will learn to write **equivalent percents, decimals, and common fractions**.

Topic A: Writing Ratios

Ratio is a **comparison** of one number or quantity with another number or quantity. Ratio shows the **relationship** between the quantities.

Ratio is pronounced "rā' shō" or it can be pronounced "rā' shē o".

You often use ratios, look at these examples:

- Scores in games are ratios. For example,
 - The Penguins won 4 to 3
 - The Canucks lost 1 to 5
- Directions for mixing can be ratios. For example,
 - Use 1 egg to each cup of milk
 - Mix 25 parts gas to 1 part oil for the motorcycle
- Betting odds are given as ratios. For example,
 - Black Jade is a 3 to 1 favourite
 - The heavyweight contender is only given a 2 to 5 chance to win
- The *scale* at the bottom of maps is a ratio. For example,
 - 1 centimetre represents 10 kilometres
- Prices are often given as ratios. For example,
 - 100 grams for \$0.79
 - 2 cans for \$1.85

For ratios to have meaning you must know **what** is being compared, and the units that are being used. Read these examples of ratios and the units that are used. A general ratio may say "parts" for the units.

- A.** It rained four days and was sunny for three days last week. The **ratio of rainy days to sunny days** was 4:3.

4:3 is properly read "**4 is compared to 3**" but is often read "**4 to 3**".

- B.** The class has 12 men and 15 women registered. The ratio of men to women in the class is 12:15.

- C. At the barbeque, 36 hot dogs and 18 hamburgers were eaten. The ratio of hot dogs eaten to hamburgers eaten is 36:18.
- D. The class spends 3 hours on English and 2 hours on math each day. The ratio of time spent on English compared to math is 3:2.

Exercise One

Write the ratios asked for in these questions, using the : symbol (eg., 4:1). Write the units and what is being compared beside the ratio.

- a) Powdered milk is mixed using 1 part of milk powder to 3 parts of water. Write a ratio to compare the milk powder to the water.

1:3 1 part of milk powder to 3 parts of water

- b) One kilogram of ground beef will make enough hamburger for 5 people. Write a ratio to express the amount of ground beef for hamburgers to the number of people.

- c) Seventy-five vehicles were checked by the police. 15 vehicles did not meet the safety standards, but 60 of them did. Write a ratio comparing the unsafe vehicles to the safe vehicles.

- d) The liquid fertilizer we use when watering our house plants uses 7 drops of fertilizer to 1 litre of water. Write the ratio of fertilizer to water.

e) The recipe says to roast a turkey according to its weight. For every kilogram, allow 40 minutes of cooking. Write a ratio comparing time to weight.

f) The 4 litre pail of semi-transparent oil stain should cover 24 square metres of the house siding if the wood is smooth. Write the ratio comparing quantity of stain to the smooth wood surface area.

g) The same 4 L of stain will only cover 16 square metres of the house siding if the wood is rough. Write that ratio.

Answers to Exercise One

- a) 1:3 1 part of milk powder to 3 parts water
- b) 1:5 1 kg of beef to 5 people
- c) 15:60 15 unsafe vehicles to 60 safe vehicles
- d) 7:1 7 drops fertilizer to 1 litre of water
- e) 40:1 40 minutes to 1 kg of turkey
- f) 4:24 4 L of stain to 24 m² of smooth wood
- g) 4:16 4 L of stain to 16 m² of rough wood

The numbers that you have been using to write the ratios are called the **terms of the ratio**.

The order that you use to write the terms is very important. You read a ratio from left to right and the order must match what the numbers mean. For example, 3 scoops of coffee to 12 cups of water must be written 3:12 as a ratio because you are comparing the quantity of coffee to the amount of water.

If you wish to talk about the amount of water compared to the coffee you have, you would say "use 12 cups of water for every 3 scoops of coffee" and the ratio would be written 12:3.

Ratios can be written 3 different ways:

- using the : symbol 2:5
- as a common fraction $\frac{2}{5}$
 - the first number in the ratio is the numerator; the second number is the denominator.
 - ratios written as a common fraction are read as a ratio, not as a fraction say "2 to 5", not two-fifths.
- using the word "to" 2 to 5

Exercise Two

Use the ratios you wrote in Exercise One to complete the chart.

	:	Common fraction	to
a)	1:3	$\frac{1}{3}$	1 to 3
b)			
c)			
d)			
e)			
f)			
g)			

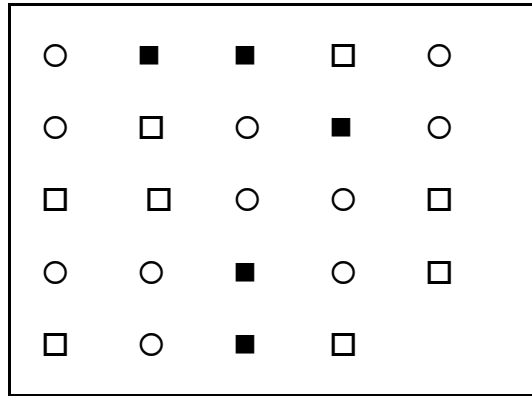
Answers to Exercise Two

- a) 1:3 or $\frac{1}{3}$ or 1 to 3 b) 1:5 or $\frac{1}{5}$ or 1 to 5 c) 15:60 or $\frac{15}{60}$ or 15 to 60
d) 7:1 or $\frac{7}{1}$ or 7 to 1 e) 40:1 or $\frac{40}{1}$ or 40 to 1 f) 4:24 or $\frac{4}{24}$ or 4 to 24
g) 4:16 or $\frac{4}{16}$ or 4 to 16

Exercise Three

Use the diagrams to write a ratio comparing the quantity of each shape, as asked.

a)



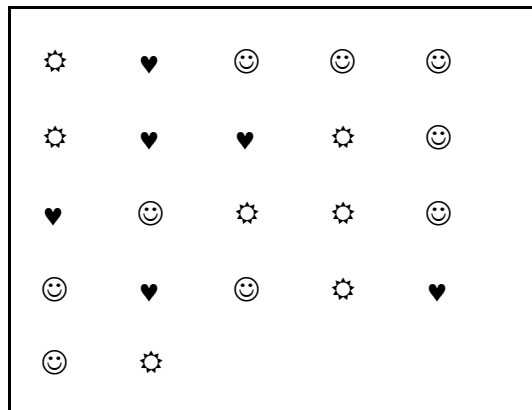
i) \square to \bigcirc _____

ii) \blacksquare to \square _____

iii) \bigcirc to \square _____

iv) \blacksquare to \bigcirc _____

b)



i) ☺ to ♥ _____

ii) ⚙ to ☺ _____

iii) ♥ to ☺ _____

iv) ⚙ to ♥ to ☺ _____

Answers to Exercise Three

a) i) 8:11 or $\frac{8}{11}$ or 8 to 11 ii) 5:8 or $\frac{5}{8}$ or 5 to 8 iii) 11:8 or $\frac{11}{8}$ or 11 to 8 iv) 5:11 or $\frac{5}{11}$ or 5 to 11

b) i) 9:6 or $\frac{9}{6}$ or 9 to 6 ii) 7:9 or $\frac{7}{9}$ or 7 to 9 iii) 6:9 or $\frac{6}{9}$ or 6 to 9 iv) 7:6:9 or 7 to 6 to 9

Equivalent Ratios

Like equivalent fractions, equivalent ratios are equal in value to each other.

$$10:100 = 1:10$$

Ratios can be written as common fractions. It is convenient to work with ratios in the common fraction form.

You can then easily:

- Find equivalent ratios in higher terms
- Find equivalent ratios in lower terms
- Find a missing term

Example A: Express 4:5 in higher terms

$$4:5 = \frac{4}{5} \longrightarrow \frac{4}{5} \left(\frac{\times 2}{\times 2} \right) \longrightarrow \frac{8}{10}$$

4:5 is equivalent to 8:10

Example B: Express 3:6 in lower terms

$$3:6 = \frac{3}{6} \longrightarrow \frac{3}{6} \left(\frac{\div 3}{\div 3} \right) \longrightarrow \frac{1}{2}$$

3:6 is equivalent to 1:2

To find equivalent ratios in higher terms, multiply each term of the ratio by the same number.

To find equivalent ratios in lower terms, divide each term of the ratio by the same number.

Exercise Four

Write equivalent ratios in any higher term. You may want to write the ratio as a common fraction first. Ask your instructor to mark this exercise.

$$\text{a) } 5:6 = \frac{5}{6} \left(\frac{\times 3}{\times 3} \right) = \frac{15}{18} = 15:18$$

$$\text{b) } 4:3 \quad \underline{\hspace{2cm}}$$

$$\text{c) } 10:2 \quad \underline{\hspace{2cm}}$$

$$\text{d) } 50:1 \quad \underline{\hspace{2cm}}$$

$$\text{e) } 9:4 \quad \underline{\hspace{2cm}}$$

$$\text{f) } 3:5 \quad \underline{\hspace{2cm}}$$

$$\text{g) } 6:2 \quad \underline{\hspace{2cm}}$$

$$\text{h) } 7:8 \quad \underline{\hspace{2cm}}$$

$$\text{i) } 3:1 \quad \underline{\hspace{2cm}}$$

$$\text{j) } 1:4 \quad \underline{\hspace{2cm}}$$

Answers to Exercise Four: See your instructor.

Exercise Five

Write these ratios in lowest terms—that is, simplify the ratios.

$$\text{a) } 4:12 = \frac{4}{12} \left(\frac{\div 4}{\div 4} \right) = \frac{1}{3} = 1:3$$

$$\text{b) } 10:5 \quad \underline{\hspace{2cm}}$$

$$\text{c) } 7:21 \quad \underline{\hspace{2cm}}$$

$$\text{d) } 20:5 \quad \underline{\hspace{2cm}}$$

e) 6:14 _____

f) 2:4 _____

g) 6:3 _____

h) 16:8 _____

i) 100:50 _____

j) 4:8 _____

Answers to Exercise Five:

Ratios written as a common fraction or using the word “to” will also be correct in this exercise.
The terms must be the same.

- a) 1:3 b) 2:1 c) 1:3 d) 4:1 e) 3:7 f) 1:2 g) 2:1 h) 2:1 i) 2:1 j) 1:2

Exercise Six

Using a colon : write a ratio in lowest terms for the information given.

- a) In the class of 25 students, only 5 are smokers. Write the ratio of smokers to non-smokers in the class. (Note—you must first calculate the number of non-smokers.)

- b) The police issued 12 roadside suspensions to drivers out of the 144 who were checked in the road block last Friday. Write the ratio of suspended drivers to the number checked.

- c) Twenty-seven students registered for the course and 24 completed it. Write a ratio showing number of completions compared to number enrolled.

- d) During an hour (60 minutes) of television viewing last night there were 14 minutes of commercials, so there were only 46 minutes of the actual program! Write the ratio of commercial time to program time.

- e) For each pair of coins, write the ratio comparing the value. (Use cents.)

i) a nickel to a dime 5:10 = 1:2

ii) a nickel to a quarter _____

iii) a nickel to a dollar _____

iv) a dime to a nickel _____

v) a dime to a quarter _____

vi) a dime to a dollar _____

vii) a dollar to a dime _____

Answers to Exercise Six

a) 1:4 b) 1:12 c) 8:9 d) 7:23

e) i) 1:2 ii) 1:5 iii) 1:20 iv) 2:1 v) 2:5 vi) 1:10 vii) 10:1

Topic A: Self-Test

Mark /12 Aim 10/12

A. Write the definitions. 3 marks

a) ratio _____

b) terms of the ratio _____

c) equivalent ratios _____

B. Write the ratios asked for in lowest terms. Use the : style. 4 marks
Then *read* the ratio.

a) The campground had three vacant campsites and 47 occupied sites. Write the ratio of occupied sites to vacant sites.

ratio: _____ read: " _____."

b) For every ten dogs in the city, only 2 have current dog licences. Write the ratio of licensed dogs to unlicensed dogs. (Find the number of unlicensed dogs first.)

ratio: _____ read: " _____."

C. Simplify these ratios. 5 marks

a) 9:12 _____

b) 6:4 _____

c) 500:1 000 _____

d) 2:9 _____

e) 35:15 _____

Answers to Topic A Self-Test

Part A

- a) A **ratio** is a comparison of one number or quantity with another number or quantity. Ratios show the relationship between the quantities or amounts.
- b) **Terms of a ratio** are the numbers used in the ratio, the **parts** of the ratio.
- c) **Equivalent ratios** – Ratios of equal value to each other.

Part B

- a) 47:3 read “47 occupied sites to 3 vacant sites”
- b) 1:4 read “1 licensed dog to 4 unlicensed dogs”

Part C

- a) 3:4 b) 3:2 c) 1:2 d) 2:9 e) 7:3

Topic B: Rates

When a ratio is used to compare two different kinds of measure (e.g. apples and oranges, or meters and hours), it is called a rate. The denominator must be 1.

Example A:

A car can drive 725 km on 55 L of gas. What is the rate in km per L?

The ratio of this is $\frac{725\text{km}}{55\text{L}}$, find the rate by making the denominator 1.

$$\text{Divide } \frac{725\left(\div 55\right)}{55\left(\div 55\right)} = \frac{13.18}{1} = 13.18$$

The rate is 13.18 km/L

Example B:

Sue bought 10 lb of oranges for \$4.99. What is the rate in cents per pound?

The ratio is $\frac{\$4.99}{10\text{lb}} = \frac{499\text{cents}}{10\text{lb}}$, find the rate by making the denominator 1.

$$\text{Divide } \frac{499\left(\div 10\right)}{10\left(\div 10\right)} = \frac{49.9}{1} = 49.9$$

The rate is 49.9 ¢/lb

When talking about rate, use the word ‘per’.

In example A, say: “The fuel economy of the car is 13.18 kilometres per litre”.

In example B, say: “The oranges cost 49.9 cents per pound”.

Example C:

It takes 60 ounces of grass seed to plant 30 m^2 of lawn. What is the rate in ounces per metre squared (m^2)?

The ratio is $\frac{60\text{oz}}{30\text{m}^2}$, find the rate by making the denominator 1.

$$\frac{60\left(\div 30\right)}{30\left(\div 30\right)} = \frac{2}{1} = 2$$

The rate is 2 oz/m^2 , or

2 ounces per square metre.

Exercise One

Write the following ratios as rates, comparing distance to time.

1) 120 km, 3 hours

2) 27 km, 9 hours

3) 203 km, 29 seconds

4) 444 km, 48 sec

Answers to Exercise One

1) 40 km/hour

2) 3 km/hour

3) 7 km/second

4) 9.25 km/second

Exercise Two

Write the following ratios as rates.

- 1) A leaky faucet can lose 52 litres of water in a week. What is the rate of litres lost per day? (round to two decimal places)

- 2) A ratio of distance travelled to time is called speed. What is the rate (speed) in kilometres per hour (km/h)?
 - a) 45 km, 3 hours _____

 - b) 129 km, 1.5 hours _____

 - c) 65 km, 13 hours _____

- 3) Vancouver Island has a population of 734 860, and a land mass of 32 134 square kilometres. What is the number of people per square kilometre? (This is called population density) Round your answer to the nearest whole number.

- 4) The population of PEI is 135 851, and its land mass is 5 660 square kilometres. What is the population density? Round your answer to the nearest whole number.
- 5) At rest, the human heart will beat an average of 4200 beats in 60 minutes. What is the rate of beats per minute?
- 6) At rest, the heart beat of an elephant is 1680 beats per 60 minutes. What is the rate of beats per minute?
- 7) At rest, the heart beat of a mouse is 30 000 beats per 60 minutes. What is the rate of beats per minute?

Answers to Exercise Two

1. 7.43 L/day 2. a. 15 km/hour b. 86 km/hour c. 5 km/hour
3. 23 people/km² 4. 24 people/km² 5. 70 beats/minute 6. 28 beats/minute 7. 500 beats/minute

Topic B: Self-Test

Mark /7 Aim 6/7

A. Write the definition.

1 mark

- a) Rate _____

B. Write the following ratios as rates.

6 marks

- a) 12 cups water, 3 cups sugar _____
- b) 72 metres, 24 seconds _____
- c) 1 365 000 people, 4000 km² _____
- d) 5000 cars on the road, 250 bikes on the road _____
- e) 12 cups of flour, 12 tsp. of baking powder _____
- f) 8 litres of gas, 2 litres of oil _____

Answers to Self-Test

1.

A rate is used when a ratio compares two different kinds of measure, and when the denominator is 1.

2.

a) 4 water/sugar

b) 3 m/second

c) 341 people/km²

d) 20 cars/bike

e) 1 cup flour/tsp baking powder

f) 4 litres gas/litre oil

Topic C: Proportion

A proportion is a statement that two ratios are equal or equivalent. Here are some proportions:

Proportion	Fraction Form	Read like this...
$1:2 = 2:4$	$\frac{1}{2} = \frac{2}{4}$	1 is to 2 as 2 is to 4
$1:4 = 25:100$	$\frac{1}{4} = \frac{25}{100}$	1 is to 4 as 25 is to 100
$18:9 = 10:5$	$\frac{18}{9} = \frac{10}{5}$	18 is to 9 as 10 is to 5
$15:20 = 3:4$	$\frac{15}{20} = \frac{3}{4}$	15 is to 20 as 3 is to 4

Proportions can be used to solve many math problems. You will soon learn to use proportions to solve problems involving percent. The techniques you practice in the next few pages are important for that problem solving work.

Problems often give incomplete information; that is, one of the terms is missing. To solve such problems, you first find the comparison or ratio that is given. It may be:

- a quantity of one thing that is mixed with a larger quantity of something else.
- a scale of measurement given on a map such as 1 cm on the map represents 100 km distance on land.
- cost for a certain number of items.
- time to travel a certain distance.

The problem will then give one term of the second ratio in the proportion. For example, if you have been told that 3 heads of lettuce cost \$1.49, you may be asked to find the cost of 7 heads of lettuce.

The missing term is the second cost. The proportion will be

$$\frac{\text{number of heads of lettuce}}{\text{cost}} = \frac{\text{number of heads of lettuce}}{\text{cost}}$$

$$\frac{3}{\$1.49} = \frac{7}{?}$$

$$3:\$1.49 = 7:?$$

The most important thing to remember is to keep the order of comparison the same in the first and second ratios in a proportion. If the first ratio compares time to distance then the second ratio in the proportion must compare time to distance.

$$\frac{\text{time}}{\text{distance}} = \frac{\text{time}}{\text{distance}}$$

or it could be

$$\frac{\text{distance}}{\text{time}} = \frac{\text{distance}}{\text{time}}$$

Once you have decided on the order of comparison it is a simple matter to write the proportion using the numbers given in the problem. Use a letter to stand for the missing term.

How would you find a missing term?

- You can use your skills with equivalent ratios (finding higher and lower terms).
- You can use your fraction skills of cross multiplying and then dividing to find the missing term.

Using Equivalent Ratios to Solve Proportions

Step 1 Decide on the order of comparison and write a ratio that describes the information given in the problem. Write a proportion using words of the items that are being compared in fraction form.

Step 2 Write two more ratios with the numbers matching the words in the first ratio. The missing term (number) can be given a letter (ex. N).

Step 3 Mentally set the ratio with words (the first ratio) aside.

Step 4 Multiply or divide the complete ratio to find the missing term.

Example A:

Use 1 teaspoon of baking powder for every 2 cups of flour. If a recipe uses 6 cups of flour, how much baking powder is needed?

The missing term is the teaspoons of baking powder for 6 cups of flour. Call this term N .

Step 1 Ratio is $\frac{\text{baking powder}}{\text{flour}}$

Step 2 $\frac{\text{baking powder}}{\text{flour}} = \frac{1}{2} = \frac{N}{6}$

Step 3 $\frac{1}{2} = \frac{N}{6}$

Step 4 $\frac{1 \left(\begin{smallmatrix} \times 3 \\ \times 3 \end{smallmatrix} \right)}{2 \left(\begin{smallmatrix} \times 3 \\ \times 3 \end{smallmatrix} \right)} = \frac{3}{6}$ so $\frac{1}{2} = \frac{3}{6}$ so $N = 3$

Use 3 teaspoons of baking powder for 6 cups of flour.

Example B:

Reports suggest that 3 out of 10 people will at some time miss work due to back pain. If a company has 1000 employees, how many can be expected to miss work due to back pain.

The missing term is the number of people out of 1000 who will miss work due to back pain. Call this term P .

$$\text{Step 1} \quad \frac{\text{people who miss work}}{\text{all the people at work}}$$

$$\text{Step 2} \quad \frac{\text{people who miss work}}{\text{all the people at work}} = \frac{3}{10} = \frac{P}{1000}$$

$$\text{Step 3} \quad \frac{3}{10} = \frac{P}{1000}$$

$$\text{Step 4} \quad \frac{3(\times 100)}{10(\times 100)} = \frac{300}{1000} \quad \text{so} \quad \frac{3}{10} = \frac{300}{1000} \quad \text{so} \quad P = 300$$

300 people out of 1 000 people may miss work due to back pain.

Exercise One

Write the ratio of the words to describe the information given.

- a) Three cup of flour to one teaspoon of yeast.

a. Example: $\frac{\text{flour}}{\text{yeast}}$

- b) Four parts oil, ten parts gasoline _____

- c) One centimetre represents 100 kilometres _____

- d) 100 grams for \$6.89 _____
- e) 3 eggs for each cup of milk _____
- f) 5 men and 7 women _____
- g) Four hours spent on cleaning and two hours spent on making food. _____

Answers to Exercise One

$\frac{oil}{gasoline}$ b) $\frac{centimetres}{kilometres}$ c) $\frac{grams}{money}$ d) $\frac{eggs}{milk}$ e) $\frac{men}{women}$ f) $\frac{cleaning}{makingfood}$ g)

Exercise Two

Use equivalent ratios to find the answers.

- a) One cup of sugar and four cups of water will make great hummingbird food. How much sugar do you need for 8 cups of water?

$$\frac{sugar}{water} \longrightarrow \frac{1}{4} = \frac{N}{8} \longrightarrow \frac{1 \left(\begin{smallmatrix} \times 2 \\ \times 2 \end{smallmatrix} \right)}{4 \left(\begin{smallmatrix} \times 2 \\ \times 2 \end{smallmatrix} \right)} = \frac{2}{8} \longrightarrow N = 2$$

- b) Reports show that for every 100 vehicles checked by police, 20 vehicles do not meet the safety standard. If only 50 vehicles are checked, how many would not meet the safety standard?

- c) Four litres of paint covers 24 square metres of wall. How much paint is needed to cover 72 square metres?
- d) Powdered milk uses 1 part milk powder to 3 parts water. How much powder should be added to 9 parts water?
- e) Five dollars will buy two litres of milk. How much milk can be bought with \$7.50?
- f) The Echinacea bottle directs to add 7 drops of Echinacea to 250 ml of water. How many drops go into 1 litre of water? (note: 1L = 1000 ml)

Answers to Exercise Two

- a) 2 cups of sugar b) 10 cars would not meet the safety standards c) 12 litres of paint
d) 3 parts milk powder e) 3 litres of milk f) 28 drops of Echinacea

Exercise Three

Use equivalent ratios to find the missing term in these proportions.

a) $3:5 = Y:15$ _____

b) $1:2 = P:8$ _____

c) $5:7 = 10:N$ _____

d) $2:3 = 8:W$ _____

e) $4:7 = 16:A$ _____

f) $1:3 = 2:N$ _____

g) $9:10 = C:50$ _____

h) $500:1000 = N:2$ _____

i) $15:20 = 3:A$ _____

j) $9:11 = 18:N$ _____

k) The KX 250 motorcycle uses a mixture of one part oil to 30 parts of gasoline. How much oil must be added to 3000 mL (3 litres) of gasoline?



Answers to Exercise Three

a) $Y = 9$

b) $P = 4$

c) $N = 14$

d) $W = 12$

e) $A = 28$

f) $N = 6$

g) $C = 45$

h) $N = 1$

i) $A = 4$

j) $N = 22$

k) $1:30 = N:3000$ $N = 100$ mL

Using Cross-Multiplication to Solve a Proportion

Review Cross Products:

$$\frac{2}{5} \quad \begin{array}{l} \nearrow \\ \searrow \end{array} \quad \frac{4}{10}$$

$$2 \times 10 = 5 \times 4$$

$$20 = 20$$

$$\text{so, } \frac{2}{5} = \frac{4}{10}$$

Remember that when the cross products are the same, the fractions are equivalent.

When finding the missing terms in a proportion, cross-multiplication can be used. Follow the examples carefully.

Example A: $\frac{2}{3} = \frac{N}{45}$

Cross-multiply $2 \times 45 = 3 \times N$

$$90 = 3N$$

The idea is to have the unknown term N by itself on one side of the equal sign. To do that, remember these things that you already know:

- division and multiplication are opposite operations
- whatever is done to one side of an equation or proportion must be done to the other side to keep the equation equal

$3N$ means N is multiplied by 3. **To get rid of the 3, divide by 3.**

You must also divide the other side of the equation by 3.

$$\frac{90}{3} = \frac{3N}{3}$$

Solve by reducing the $\frac{3}{3}$ and by dividing 90 by 3.

$$\frac{90}{3} = \frac{3N}{3}$$

$$\frac{90}{3} = \frac{1N}{1}$$

$$\frac{90}{3} = N$$

$$90 \div 3 = N$$

$$30 = N$$

Reducing the fraction $\frac{3N}{3}$ to $\frac{1N}{1}$ to N is also called *cancelling*. In math, a fraction can be cancelled when the numerator and denominator are the same number.

$$\text{e.g.: } \frac{6P}{6} = \frac{1P}{1} = P$$

Example B: $6:7 = 24:N$

$$\frac{6}{7} = \frac{24}{N}$$

Cross-multiply $6 \times N = 7 \times 24$
 $6N = 168$

Divide both sides by 6. The 6's with the N will cancel (reduce), and the N will be alone.

$$\frac{\cancel{6}N}{\cancel{6}} = \frac{168}{6}$$

$$N = 168 \div 6$$
$$N = 28$$

$$\frac{6}{7} = \frac{24}{28}$$

Check by cross-multiplying

$$\text{Is } 6 \times 28 = 7 \times 24?$$

$$6 \times 28 = 168$$

$$7 \times 24 = 168$$

the cross-product $168 =$ the cross product 168

Yes $6:7 = 24:28$

Example C: $8:10 = N:80$

$$\frac{8}{10} = \frac{N}{80}$$

Cross-multiply $8 \times 80 = 10 \times N$
 $640 = 10N$

Divide both sides by 10 so N will be alone.

$$\frac{640}{10} = \frac{10N}{10}$$

$$64 = N$$

To Solve a Proportion Problem Using Cross-Multiplication

Step 1 Write the first ratio using the information given.

Step 2 Write the proportion, using a letter in place of the missing term. Be sure the **order of comparison is the same** in both the first and second ratios in your proportion.

Step 3 Write the proportion in the fraction form. (Try to **simplify** the ratio **before** you do all the calculations.)

Step 4 Cross-multiply and set the cross-products equal to each other.

Step 5 Divide both sides of the equation by the number with the unknown term.

Step 6 Check by putting your answer back into the original proportion and cross-multiplying.

Exercise Four

Practise using cross-multiplying to find the missing term in these proportions.

$$\text{a) } \frac{5}{8} = \frac{N}{32}$$

$$\text{b) } \frac{4}{N} = \frac{24}{30}$$

$$5 \times 32 = 8 \times N$$

$$160 = 8N$$

$$\frac{160}{8} = \frac{8N}{8}$$

$$160 \div 8 = N$$

$$20 = N$$

$$\text{c) } \frac{12}{4} = \frac{18}{x}$$

$$\text{d) } \frac{y}{6} = \frac{20}{12}$$

e) $4:15 = 8:N$

f) $W:100 = 6:50$

g) $20:N = 5:4$

h) $5:12 = 25:x$

i) $6:9 = y:3$

j) $16:B = 8:3$

Answers to Exercise Four

b) $N = 5$

c) $x = 6$

d) $y = 10$

e) $N = 30$

f) $W = 12$

g) $N = 16$

h) $x = 60$

i) $y = 2$

j) $B = 6$

The numbers in a ratio often are common fractions, decimals or mixed numbers. Follow exactly the same steps that you have been using to solve whole number proportions. The calculations will use your skills with fractions.

Example A: $2\frac{1}{4} : 3 = N : 7$

Rewrite the proportion $\frac{2\frac{1}{4}}{3} = \frac{N}{7}$
 Cross-multiply

$$2\frac{1}{4} \times 7 = 3 \times N$$

$$\frac{9}{4} \times \frac{7}{1} = 3 \times N$$

$$\frac{63}{4} = 3N$$

$$\frac{63}{4} \div \frac{3}{1} = \frac{3 \times N}{3} \longrightarrow \frac{63}{4} \times \frac{1}{3} = N$$

$$\frac{63}{12} = N \longrightarrow 5\frac{1}{4} = N$$

Exercise Five

Solve these proportions.

a) $6.5 : 5 = 13 : A$

$$\frac{6.5}{5} = \frac{13}{A}$$

$$6.5A = 65$$

$$A = 65 \div 6.5$$

$$A = 10$$

b) $3\frac{1}{2} : 2 = N : 8$

c) $9:6 = 4\frac{1}{2} : N$

d) $7.5:B = 10:20$

e) $3.75:5 = 9:x$

f) $4\frac{1}{8} : A = 3:6$

g) $Y : 1\frac{1}{2} = 12:4$

h) $6.125 : 24.5 = 4:N$

Answers to Exercise Five

b) $N = 14$

c) $N = 3$

d) $B = 15$

e) $x = 12$

f) $A = 8\frac{1}{4}$ or 8.25

g) $Y = 4\frac{1}{2}$ or 4.5

h) $N = 16$

Exercise Six

Solve these problems by proportion. Remember that the **order of comparison** must be the same in both ratios of a proportion.

a) Joanne can walk 18 km in 3 hours. How far can she walk, at the same rate in $5\frac{1}{2}$ hours?

b) The taxes on the property valued at \$300 000 are valued at \$5000. At the same rate of taxation, what would the taxes be on the smaller lot down the street which is valued at \$240 000?

c) Liquid fish fertilizer smells awful but does a wonderful job! The concentrated fertilizer is mixed 15 millilitres to 4 litres of water. How much fertilizer would you mix with the 10 litre watering can of water?

d) One B.C. road map has a scale of 0.5 centimetres equal to 10 kilometres. Complete the chart by calculating actual driving distances in kilometres between some B.C. places.

The proportions will be $\frac{0.5}{10} = \frac{\text{cm given in chart}}{\text{actual distance in km}}$

Places in B.C.	Number of cm between places on the map	Actual distance in kilometres
Kelowna and Vernon	2.5 cm	
Burns Lake and Vanderhoof	5.5 cm	
TaTa Creek and Skookumchuk	0.75 cm	
Kitimat and Terrace	3.3 cm	

e) The directions on the lawn fertilizer say to spread 1.7 kg over 100 m² of lawn.

i) How much fertilizer is needed for a 130 m² lawn?

ii) How much fertilizer for a 75 m² lawn?

f) To mix concrete, combine 1 part Portland cement, 5 parts aggregate (a sand and gravel mixture) and $\frac{1}{2}$ part water. People often use a shovel for 1 part.

You want to use 3 shovels (parts) of cement.

i) How much aggregate will you need?

ii) How much water will you need?

Answers to Exercise Six

a) 33 km

b) \$4000

c) 37.5 mL

d)

Places in B.C.	Number of cm between places on the map	Actual distance in kilometres
Kelowna and Vernon	2.5 cm	50 km
Burns Lake and Vanderhoof	5.5 cm	110 km
TaTa Creek and Skookumchuk	0.75 cm	15 km
Kitimat and Terrace	3.3 cm	66 km

e) i) 2.21 kg

ii) 1.275 kg

f) i) 15 shovels aggregate ii) 1.5 shovels of water

A. Solve these proportions.**6 marks**

a) $N:14 = 28:56$

b) $3:11 = N:22$

c) $50:45 = 10:N$

d) $4\frac{1}{5}:Y = 3:2$

e) $12:4 = N:7$

f) $7.75:N = 12:6$

B.**14 marks**

a) Get a map of BC, a map of Canada and a map of your city or town.

b) Find the scale on each map (usually at the bottom) and write down the ratio of map distance to the actual distance.

c) With another student or an instructor, calculate actual distances between places by measuring the distance on the map and working out the proportion according to the scale given. Do at least three distance calculations on each map.

Ask your instructor to mark your work.

Answers to Topic C Self-Test

Part A

a) $N = 7$

b) $N = 6$

c) $N = 9$

d) $Y = 2\frac{4}{5}$ or 2.8

e) $N = 21$

f) $N = 3.875$

Part B – See your instructor.

Unit 1 Review

1. Write the ratios asked for in the questions. Reduce when needed.
 - a) It snowed for two days and was sunny for five days last week. Write a ratio to compare snowy days to sunny days.

 - b) The class spends 6 hours a week on math and 8 hours on English. Write the ratio of hours spent on math to English.

 - c) The chickens at Glen's farm laid 3 eggs on Tuesday and 5 eggs on Friday. Write the ratio of numbers of eggs laid.

 - d) The perimeter of a room is 14 metres, and the length is 3.5 metres. Write a ratio comparing the two numbers.

 - e) One kilogram of pork will make enough sausages for 7 people. Write a ratio to express the amount of pork for sausages to the number of people.

 - f) A jam recipe calls for 8 cups of blue berries to 3 cups of sugar. Write a ratio to express the amount of berries to sugar.

2. Write the ratios in lowest terms.

a) 2:5 _____ b) 20:5 _____

c) 14:21 _____ d) 2:4 _____

e) 8:24 _____ f) 15:150 _____

g) 130:26 _____ h) 9:54 _____

i) 6:3 _____ j) 6:24 _____

3. Write the following ratios as rates.

a) 16 cups of water to 4 cups of sugar _____

b) 150 kilometres to 1.25 hours _____

c) 12 cups of flour to 3 eggs _____

d) 465 000 people to 3000 square kilometres _____

e) 35 kilometres to 7 hours _____

f) \$3.99 to 10 pounds of apples _____

4. Use cross multiplication to solve the proportions.

a) $\frac{5}{8} = \frac{P}{24}$

b) $\frac{S}{100} = \frac{8}{25}$

c) $\frac{4}{N} = \frac{8}{40}$

d) $\frac{4}{17} = \frac{F}{68}$

d) $\frac{8}{30} = \frac{12}{N}$

e) $\frac{6}{9} = \frac{A}{36}$

5. Use cross multiplication to solve the proportions.

a) $\frac{2\frac{1}{4}}{5} = \frac{A}{30}$

b) $\frac{L}{7\frac{1}{2}} = \frac{40}{30}$

$$c) \frac{11}{8} = \frac{4\frac{1}{8}}{P}$$

$$e) \frac{5}{34} = \frac{Y}{1\frac{1}{2}}$$

$$d) \frac{3\frac{3}{4}}{5} = \frac{8}{X}$$

$$f) \frac{2\frac{2}{3}}{3} = \frac{A}{16}$$

Answers to Unit 1 Review

1.

- a) 2:5 b) 6:8 c) 3:5 d) 14:3.5
 e) 1 kg : 7people f) 8 cups berries : 3 cups sugar

2.

- a) 2:5 b) 4:1 c) 2:3 d) 1:2
 e) 1:3 f) 1:10 g) 5:1 h) 1:6
 i) 2:1 j) 1:4

3.

- a) 4 cups water per cup of sugar b) 120 km/hour
 c) 4 cups flour per egg d) 155 people per km²
 e) 5 km/hour f) \$0.399/pound or 39.9¢/pound

4.

- a) 15 b) 32 c) 20 d) 16
 e) 45 f) 24

5.

- a) $13\frac{1}{2}$ b) 10 c) 3 d) $10\frac{2}{3}$
 e) $\frac{15}{68}$ f) $14\frac{2}{9}$

TEST TIME!

Ask your instructor for the Practice Test for this unit.

Once you've done the practice test,
you need to do the unit 1 test.

Again, ask your instructor for this.

Good luck!

